Problem 2.21 (Engel)

Show that the functions $\phi_n(\theta) = e^{in\theta}$ form an orthogonal set if n is an integer (and θ ranges from $0 \to 2\pi$)

Strategy. Inner products involving orthogonal functions vanish. In other words, I need to show:
$\int_0^{2\pi} \phi_n{}^*(\theta)\ \phi_m(\theta)\, d\theta = 0$ when n≠m.

Execution.
$\int_0^{2\pi} \phi_n{}^*(\theta)\, \phi_m(\theta)\, d\theta =$

$$= \int_0^{2\pi} e^{-in\theta}\, e^{im\theta}\, d\theta$$

$$= \int_0^{2\pi} e^{i(m-n)\theta}\, d\theta$$

$$= \frac{e^{i(m-n)\theta}}{i(m-n)\theta} = \frac{\cos(m-n)\theta + i\sin(m-n)\theta}{i(m-n)\theta}$$ (evaluate at $\theta = 2\pi$ & 0)

sin(m-n)θ = 0 when $\theta = 2\pi$ & 0
cos(m-n)θ = 1 when $\theta = 2\pi$ & 0

Therefore, the integral works out to:

$$= \frac{1 + i(0) - 1 - i(0)}{i(m-n)(2\pi - 0)} = 0$$

Notice that the ratio is only defined when m-n ≠ 0, i.e., when m≠n.

$$\int_0^{2\pi} e^{-in\theta}\, e^{im\theta}\, d\theta$$

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0
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