Generally useful constants (MKS):

```
amu = 1.6605 (10^-27) kg
NA = 6.02221 (10^23) / mole
k = 1.38066 (10^-23) J / K
me = 9.1094 (10^-31) kg
h = 6.62608 (10^-34) J s
q = 1.6022 (10^-19) C
c = 2.9979 (10^8) m / s
```

1.6605×10^{-27} kg

$\frac{6.02221 \times 10^{23}}{\text{mole}}$

$\frac{1.38066 \times 10^{-23}\ \text{J}}{\text{K}}$

9.1094×10^{-31} kg

6.62608×10^{-34} J s

1.6022×10^{-19} C

$\frac{2.9979 \times 10^{8}\ \text{m}}{\text{s}}$

Problem 2.3 (Engel)

A plot of the wave function $\psi(x,t) = A \sin(kx-\omega t)$ as a function of either x or t requires that one variable be set at a constant value, x_0 or t_0.

Part A.

If $\psi(x_0, 0) / \psi_{max} = -0.280$, what is the value of x_0 consistent with the plot shown in the upper panel of Figure 2.3?

Solution

Strategy. The problem asks about $\psi(x_0, 0) / \psi_{max}$. Inspection of Figure 2.3 shows $\psi_{max} = 1$. It is important to realize that ψ_{max} corresponds to A in $\psi = A \sin(\text{kx} - \omega \text{t})$.

The question asks about $t = 0$, and the expression $(\text{kx} - \omega\text{t})$ simplifies to kx. Therefore, the task is to find kx_0 such that $\sin(\text{kx}_0) = -0.280$.

There are two unknowns in this equation, k and x_0, but the figure caption states that $\lambda = 1.46$ m so k is actually known by

$$k = \frac{2\pi}{\lambda}$$

Execution. First, calculate k

```
λ = 1.46 m
kk = 2 π / λ

1.46 m
```

```
4.30355
-------
   m
```

Since $\sin(\text{k}x_0) = -0.280$, then $\text{k}x_0 = \arcsin(-0.280)$. I use this relationship to calculate $\text{k}x_0$ and I use $\arcsin(-0.280) = \pi - \text{kx}_0$ to generate the second value of $\text{k}x_0$.

```
kx0 = {ArcSin[-0.280], π - ArcSin[-0.280]}

{-0.283794, 3.42539}
```

Solving for x_0

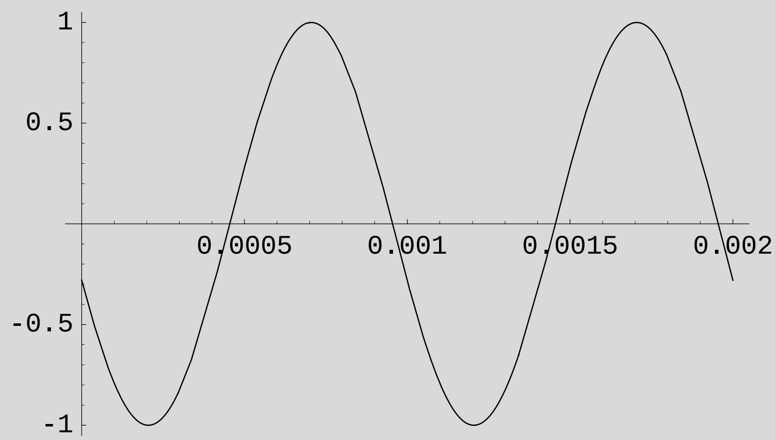
```
x0 = kx0 / kk

{-0.0659442 m, 0.795944 m}
```

Which value of x_0 is correct, i.e., corresponds to the graph in Figure 2.3?

I can determine this by plotting sin(kx_0- 2πt/T) for both values of kx_0 and comparing the result to the graph in Figure 2.3. Notice that the Figure caption says T = 0.001 s, and the graph runs from $0 \leq t < 0.002$ (the book says $10^3\, t$).

If kx_0 = -0.284

```
Plot[Sin[-0.284 - (2 π t / .001)], {t, 0, .002}]
```

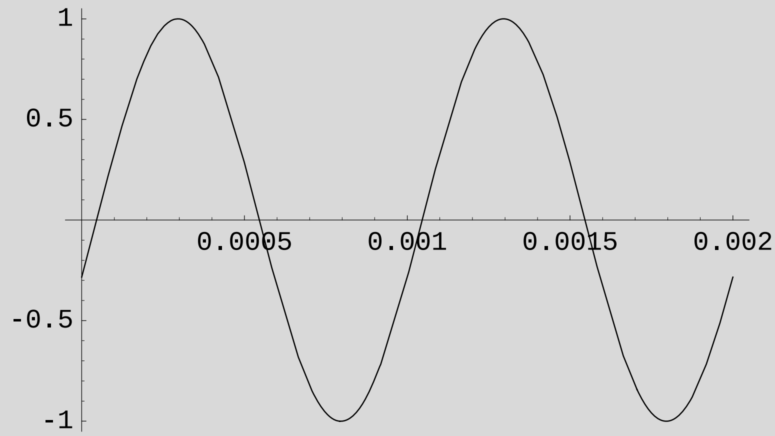



```
- Graphics -
```

This graph starts out at the right amplitude, but heads in the wrong direction (downward).

If kx_0 = 3.43

```
Plot[Sin[3.43 - (2 π t / .001)], {t, 0, .002}]
```




```
- Graphics -
```

This graph agrees with Figure 2.3 (top), so the second value, x_0 = 0.796 m, is correct.

Part B.

If $\psi(0, t_0) / \psi_{max}$ = -0.309, what is the value of t_0 consistent with the plot shown in the lower panel of Figure 2.3?

Solution

Strategy. The solution proceeds in exactly the same way as in part A. I use arcsin to generate two possible solutions for $-\omega t_0$ and then t_0. Next, I generate two graphs and compare the results with the graph in Figure 2.3.

Execution. First I generate two possible solutions of $-\omega t_0$ and t_0. The Figure caption states that T = 0.001 s, and T is related to ω by $\omega = 2\pi\nu = 2\pi/T$.

```
ωt0 = -{ArcSin[-0.309], (π - ArcSin[-0.309])}

{0.314141, -3.45573}
```

```
T = 0.001 s
ω = 2 π / T
t0 = ωt0 / ω

0.001 s
```

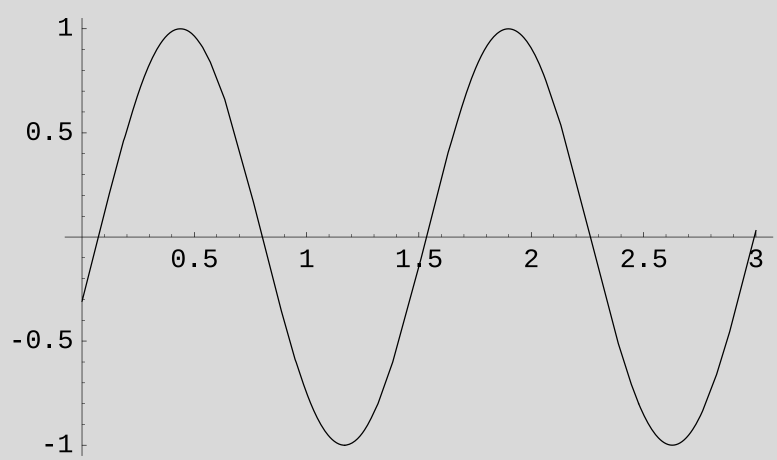
```
6283.19
───────
   s
```

```
{0.0000499972 s, -0.000549997 s}
```

I can generate two possible graphs of sin($2\pi x/\lambda$ - ωt_0). Notice that the Figure caption states that λ=1.46 m, and the graph runs from $0 \le x \le 3$ m.

We try ωt_0 = 0.314 first:

```
Plot[Sin[(2 π x / 1.46) - 0.314], {x, 0, 3}]
```

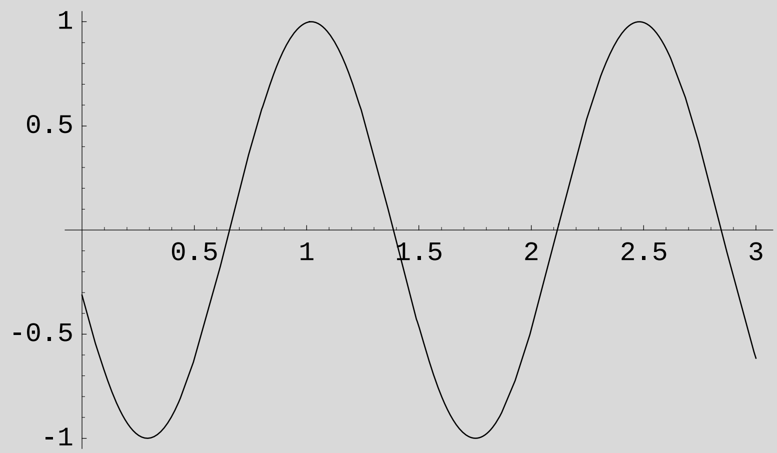



```
- Graphics -
```

This graph starts at the right amplitude, but it heads in the wrong direction (upwards).

If ωt_0 = -3.46

```
Plot[Sin[(2 π x / 1.46) + 3.46], {x, 0, 3}]
```




```
- Graphics -
```

This graph agrees with Figure 2.3 (top), so the second value, t_0 = -0.00055 s, is correct.