Problem 2.9 (Engel)

Using cos $\theta = (e^{i\theta} + e^{-i\theta})/2$ and sin $\theta = (e^{i\theta} - e^{-i\theta})/2i$, show the following

A. $\cos^2\theta + \sin^2\theta = 1$

Solution.

$\cos^2\theta =$

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Expand[((e^(iθ) + e^(-iθ))/2)^2]
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$$\frac{1}{2} + \frac{e^{-2i\theta}}{4} + \frac{e^{2i\theta}}{4}$$

$\sin^2\theta =$

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Expand[((e^(iθ) - e^(-iθ))/(2 i))^2]
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$$\frac{1}{2} - \frac{e^{-2i\theta}}{4} - \frac{e^{2i\theta}}{4}$$

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(1/2 + e^(-2iθ)/4 + e^(2iθ)/4) + (1/2 - e^(-2iθ)/4 - e^(2iθ)/4)
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$$1$$

B. $\frac{d\cos\theta}{d\theta} = -\sin\theta$

Solution.

$$\frac{de^{i\theta}}{d\theta} = ie^{i\theta}$$

$$\frac{de^{-i\theta}}{d\theta} = -ie^{-i\theta}$$

$$\frac{d\cos\theta}{d\theta} = \frac{ie^{i\theta} - ie^{-i\theta}}{2}\,\frac{i}{i} = \frac{-e^{i\theta} + e^{-i\theta}}{2i} = -\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right) = -\sin\theta$$

C. $\sin(\theta + \frac{\pi}{2}) = \cos\theta$

Solution.

$$\sin(\theta + \tfrac{\pi}{2}) = \frac{1}{2i}(e^{i(\theta+\pi/2)} - e^{-i(\theta+\pi/2)})$$

$$= \frac{1}{2i}(e^{i\theta}\, e^{i(\pi/2)} - e^{-i\theta}\, e^{-i(\pi/2)})$$

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ⅇ^(ⅈ (π/2))
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ⅈ

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ⅇ^(-ⅈ (π/2))
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−ⅈ

$= \frac{1}{2i}\, i(e^{i\theta} + e^{-i\theta})$

$= (e^{i\theta} + e^{-i\theta})/2 = \cos\theta$