Problem 2.29 (Engel)

Part A.

What are the results of these matrix multiplications (note: although the text shows 1x2 row vectors, it explicitly says "column vectors" so I have changed these vectors into 2x1 column vectors:

A.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0\,(1) + 1\,(0) \\ 1\,(1) + 0\,(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Each element of the "product" matrix is the "dot product" of one row in the *left* matrix and a column in the *right* matrix. See if you can recognize this idea in this problem and the following ones.

B.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0\,(0) + 1\,(1) \\ 1\,(0) + 0\,(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

C.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0\,(1) + 1\,(1) \\ 1\,(1) + 0\,(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

D.

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0\,(-1) + 1\,(1) \\ 1\,(-1) + 0\,(1) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Part B.

Referring to the matrix and column vectors in Part A, think of the matrix as an *operator*, and think of the column vectors as *wave functions*. The action of the operator on the wave function is simply the matrix-vector multiplication carried out in Part A. Given these definitions, are any of the wave functions in Part A eigenfunctions of the operator?

The answer for **D.** is shown in detail below

$$\hat{O} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\psi = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\hat{O}\psi = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -1\,\psi$$

This wave function is an eigenfunction of the operator with eigenvalue = -1. (Nomenclature note: It is acceptable to say, "this vector is an eigenvector of the matrix with eigenvalue = -1".)

For **A-C**: **A - not** an eigenfunction, **B - not** an eigenfunction, **C -** eigenfunction with eigenvalue = 1.