Problem 4.3 (Engel)

Are the total energy eigenfunctions for the free particle in one dimension, $\psi^+(x) = A_+ e^{+i k x}$ and $\psi^-(x) = A_- e^{-i k x}$ where $k = \sqrt{(2 m E)/\text{hbar}^2}$, also eigenfunctions of the one-dimensional linear momentum operator? If so, what are the eigenvalues?

Solution

Strategy. Apply the linear momentum operator, $\hat{p}_x = -i\, \text{hbar}\, \partial / \partial x$, to each eigenfunction of the total energy operator. To be an eigenfunction of $\hat{p}_x$ the result must be a scalar multiple of the original function.

Execution.

Note that we can ignore the normalization constants A_+ and A_-. Note too that k is not a function of x, so we can use k in our work instead of the more complicated formula.

```
Clear[x, k]
ψplus[x_] := ⅇ^(ⅈ k x)
∂x ψplus[x]

ⅈ ⅇ^(ⅈ k x) k
```

Clearly, $\psi^+(x)$ is an eigenfunction of $\hat{p}_x$. To get its eigenvalue, we multiply the differential by $-i$ hbar.

```
eigenvalueplus = % (- ⅈ hbar) / ψplus[x]

hbar k
```

Similar work shows $\psi^-(x)$ is also an eigenfunction with eigenvalue $-\text{hbar}\, k$.

```
Clear[x, k]
ψminus[x_] := ⅇ^(-ⅈ k x)
∂x ψminus[x]

-ⅈ ⅇ^(-ⅈ k x) k
```

```
eigenvalueminus = % (- ⅈ hbar) / ψminus[x]

-hbar k
```

Comment. The two energy eigenfunctions are momentum eigenfunctions too. The functions correspond to particles moving with identical momenta, but in opposite directions.