
Problem 4.5 (Engel)

Consider a particle in a one-dimensional box defined by $V(x) = 0$, $a > x > 0$ and $V(x) = \infty$, $x \geq a$ or $x \leq 0$. Explain why each of the four listed unnormalized functions is, or is not, an acceptable wave function based on criteria such as being consistent with the boundary conditions, and with the association of $\psi^*(x)\psi(x) dx$ with probability.

Solution

Part A.

$$\psi = A \cos\left(\frac{n\pi x}{a}\right)$$

Execution. An acceptable wave function must equal 0 at $x = 0$ and $x = a$. This function equals A at $x = 0$. It equals $+A$ or $-A$ at $x = a$. Therefore it is unacceptable.

Part B.

$$\psi = B(x + x^2)$$

Execution. This function equals 0 at $x = 0$ as required. However, this function equals $B(a + a^2)$ at $x = a$, which makes it unacceptable.

Part C.

$$\psi = Cx^3(x - a)$$

Execution. This function satisfies both of the boundary conditions. Its value is defined over the entire interval, so it can be used to define a probability. This is an acceptable function.

Part D.

$$\psi = \frac{D}{\sin(n\pi x/a)}$$

Execution. This function blows up at the boundaries (the denominator becomes zero). It is not acceptable because it does not satisfy the boundary conditions, and it cannot be integrated.