Problem 4.6 - modified (Engel)

Evaluate the normalization integral for the eigenfunctions of $\hat{H}$ for the particle in a box $\psi_n(x) = A\sin(n\pi x/a)$ using the trigonometric identity $\sin^2 y = (1 - \cos 2y)/2$.

My modification: don't bother with the trig identity. However, try two integrations. One with $n = 3$, and one with $n = n$.

Solution

Strategy.

The normalization integral is defined as $\int \psi^* \psi \, dx$. If the function is normalized, this integral will equal one.

Execution. ψ is real, so the normalization integral is

$$\int \psi^2 \, dx = \int A^2 \sin^2(n\pi x/a) \, dx$$

If $n = 3$

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∫_0^a Sin[3π x/a]^2 ⅆx
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$$\frac{a}{2}$$

If $n = n$

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∫_0^a Sin[n π x / a]^2 ⅆx
Simplify[%, n ∈ Integers]
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$$\frac{1}{4} a \left(2 - \frac{\text{Sin}[2 n \pi]}{n \pi}\right)$$

$$\frac{a}{2}$$

Both results agree (which is to be expected since the integral does not depend on the precise value of n).

The normalization integral simplies to:

$$= \frac{a A^2}{2}$$

and the function will be normalized if $A = \sqrt{\frac{2}{a}}$.

Comment. The normalization constant is the same for all of the eigenfunctions. It does not depend on n.