Problem 4.8-modified (Engel)

Calculate the probability that a particle in a one-dimensional box of length *a* is found between 0 and 0.25*a*, and the probability that it is found between 0.25*a* and 0.5*a*.

Part A. $n = 1$, $\psi(x) = \sqrt{\frac{2}{a}}\ \text{Sin}[\frac{\pi x}{a}]$
Part B. $n = 3$, $\psi(x) = \sqrt{\frac{2}{a}}\ \text{Sin}[\frac{3\pi x}{a}]$
Part C (my add'n). $n = 10$, $\psi(x) = \sqrt{\frac{2}{a}}\ \text{Sin}[\frac{10\pi x}{a}]$

Solution

Strategy.
Probability for finding particle inside a portion of the box is obtained by evaluating the *normalization* integral over this portion of the box.

Execution.

$$n = \{1, 3, 10\}$$

$$\text{dens} = \left(\sqrt{\frac{2}{a}}\ \text{Sin}\left[\frac{n\,\pi\,x}{a}\right]\right)^2$$

$$\text{Pleft} = \int_0^{0.25\,a} \text{dens}\ dx$$

$$\text{Pctr} = \int_{0.25\,a}^{0.5\,a} \text{dens}\ dx$$

$$\{1, 3, 10\}$$

$$\left\{\frac{2\,\text{Sin}\left[\frac{\pi x}{a}\right]^2}{a}, \frac{2\,\text{Sin}\left[\frac{3\pi x}{a}\right]^2}{a}, \frac{2\,\text{Sin}\left[\frac{10\pi x}{a}\right]^2}{a}\right\}$$

$$\{0.0908451, 0.303052, 0.25\}$$

$$\{0.409155, 0.196948, 0.25\}$$

Comment. Classical physics predicts that the particle can be found with equal probability at all points in the box. Since the left quarter and center-left quarters of the box encompass equal distances (0.25 *a*), the probability of finding the particle in either zone should be the same (0.25 *a*/*a* = 0.25).

The ground state, $n = 1$, gives very different probabilities: 0.09 and 0.41 for the left and left-center regions, respectively.

These probabilities *reverse* in the second excited state, $n = 2$: 0.30 and 0.20 for the left and left-center regions. Note that both are converging on 0.25.

The probabilities in the ninth excited state, $n = 10$, match the classical values. This blending of quantum physics into classical physics at high n is an example of Bohr's Correspondence Principle.