Economics 304 Fall 2009

Homework Set #4 Solutions

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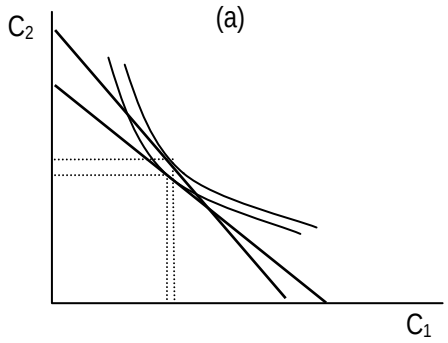
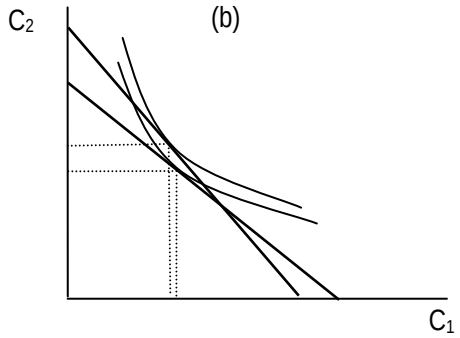
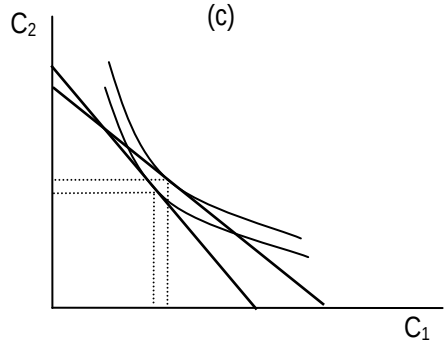
1. Draw a budget-constraint/indifference-curve diagram to illustrate each of the following situations with current consumption (C_1) on the horizontal axis and future consumption (C_2) on the vertical axis. You should assume that the person is approximately a consumption smoother, so that whether she is initially a saver or dissaver is determined mainly by her endowment point. Be sure to identify clearly the endowment point, the budget constraints before and after the change, and the amounts saved or dissaved before and after the change.

   a. A person who is initially a saver in period 1 and who responds to an increase in the real interest rate by decreasing saving.
   b. A person who is initially a saver in period 1 and who responds to an increase in the real interest rate by increasing saving.
   c. A person who is initially a dissaver in period 1 and who responds to an increase in the real interest rate by increasing saving (reducing dissaving).
   d. Explain (based on substitution and income effects) why the fourth case (dissaver who increases dissaving when the real interest rate rises) is impossible.







The situation in (d) cannot happen if consumption in each period is a normal (not inferior) good. The substitution effect will always decrease C_1. If it is a normal good, the income effect will also decrease C_1, thus saving must increase.

2. Explain why the real interest rate rather than the nominal interest rate is used in the budget constraint for consumers. Explain why the real interest rate rather than the nominal interest rate is relevant for firms deciding whether to invest in real capital.

Consumers are balancing the utility from current goods against that from future goods, so the relevant interest rate measures the opportunity cost of current goods in terms of future goods. The real interest rate does this by correcting the nominal (dollar) interest rate for changes in the purchasing power of the dollar.

Similarly, the capital investment decision involves balancing current goods (the capital goods being purchased) against future goods (the goods produced using the capital). Thus, capital investment should also depend on the real interest rate.

3. Consider a model in which individuals live for three periods, earning income of Y_1, Y_2, and Y_3 and consuming C_1, C_2, and C_3. You may assume that they receive and leave no bequests.

a. Show the equation for the individual's lifetime budget constraint.
b. Solve for permanent income using an extension of equation (8.2) in the textbook.
c. Assuming for simplicity that the real interest rate is zero and that the consumer smoothes consumption perfectly, what consumption level will she choose in each year if Y_1 = \$20,000, Y_2 = \$100,000, and Y_3 = \$0 and she has access to perfect capital markets? In an economy comprising many such consumers, what pattern of borrowing and lending will emerge among individuals of various ages?
d. How would her consumption in each period be different if the government had a program giving a \$30,000 transfer payment to all young people, paid for by a \$30,000 tax on all middle-aged people?
e. How would her consumption in each period be affected if the government had a Social Security program that paid old people \$30,000 and taxed middle-aged people \$30,000 to pay for it?
f. Re-do parts c through e under the assumption that young individuals are unable to borrow due to imperfections in the capital market.
g. What implications does your analysis have for government transfer policies?

a. $C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} = Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2}.$

b. $$Y^P + \frac{Y^P}{1+r} + \frac{Y^P}{(1+r)^2} = Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2},$$

$$Y^P\left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}\right] = Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2},$$

$$Y^P = \frac{Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2}}{1 + \frac{1}{1+r} + \frac{1}{(1+r)^2}}.$$

c. If she smoothes consumption, she will choose $C_1 = C_2 = C_3 = C = Y^P$. With the zero interest rate, all of the denominators of the present-value expressions are one, so consumption in each period is

$$C = \frac{Y_1 + Y_2 + Y_3}{3} = \frac{120,000}{3} = 40,000.$$

The individual will borrow 20,000 when young, repay this and save 40,000 when middle-aged, and dissave this 40,000 when old.

d. Her young-period income rises to 50,000 but her middle-aged income falls to 70,000. The transfer income will be exactly balanced by the tax payment, leaving lifetime income and, therefore, consumption unchanged.

e. Again, the transfer income and the tax payment will offset in the budget constraint, so lifetime income and consumption will still be 40,000. She will save 30,000 less in period two corresponding to the reduction in income and dissave 30,000 less in period three.

f. For part (c), the individual now cannot borrow when young, so her consumption is limited to 20,000. This leaves 100,000 of income in the second period to spread over consumption in periods two and three, so she consumes 50,000 in each of these periods. She is unable to smooth consumption due to the liquidity constraint imposed by imperfect capital markets. For part (d), her first-period income rises to 50,000, which means that she no longer needs to borrow in order to smooth consumption. Thus she can now consume 40,000 in each period. The transfer program increases her spending by 20,000 in the first period and reduces it by 10,000 in each subsequent period. For part (e), there is no effect of the transfer program because households were able to smooth consumption between periods two and three already. Because there is no additional income in the first period, the liquidity constraint is still binding and the household can only consume 20,000 in that period. The reduced income in period two will lower saving from 50,000 to 20,000, with a corresponding reduction in dissaving in period three.

g. Government inter-generational transfer programs have effects on consumption behavior only to the extent that they mitigate liquidity constraints. If the transfers permit households who previously could not to balance their consumption, then they will have consumption effects. Otherwise consumption-smoothing households will offset the effects of the transfers and taxes by changing their saving behavior. The MPC out of such perfectly offset disposable-income changes will be zero.

4. Read the article "Did the 2008 Tax Rebates Stimulate Spending?" *American Economic Review* 99 (2):374–379 (May 2009). (This is available through the Reed Library Web site. Search for the journal under "Print and E-Journals", then look up the issue under the "Atypon Link" selection.) What are the authors' results? Are they consistent with modern consumption theory? Are they consistent with Keynes's consumption theory? Explain your conclusions.

The authors find that most of the recipients of the tax rebates planned to save them, either by increasing their financial assets or by reducing their debts. They estimate the MPC out of these rebates to be approximately 30%. This is roughly consistent with modern consumption theory, which predicts that most households will smooth any changes in income over lifetime consumption. It is a smaller MPC than most Keynesian economists would have predicted. Note that a "pure" modern consumption model might predict a smaller MPC. If the household lives forever (dynastically) then it can only consume the real interest on any one-time increase in income if it smoothes consumption, so we would expect an MPC of 3% or so. Even if the consumption-smoothing household plans for a finite lifetime, it can increase consumption beyond the real interest rate only by $1/N$ where N is life expectancy. Thus, if the average household has a life expectancy of, say, 30 years, then the MPC should still be well under 10%. The difference between the authors' finding of 30% and the prediction of 10% can be easily explained by two extensions to the theory. First, as we have seen, households who are liquidity constrained (who want to borrow to increase consumption but can't) will have an MPC of 100%, so the fraction of liquidity-constrained households will pull the average MPC upward. Second, consumption smoothing may take the form of investment in durable goods such as autos. Since the "consumption" measure that the authors use is purchases, not use, the entire auto is counted as consumption in the period in which it is purchased. This will increase the measured MPC if households time durable-goods purchases to coincide with periods of high income.

5. Consider the theories of investment described in the text.
   a. Explain why an increase in the real interest rate would affect the desirability of investment in capital by a firm that borrows to finance its investment.
   b. Explain why (assuming perfect capital markets) the cost of capital for a firm that finances its investment y using retained earnings would be affected in the same way as if it borrowed.
   c. Explain why the increase in the real interest rate would affect Tobin's q and, therefore, the desirability of investment for a firm that finances its investment through issuing new stock.

a. If the firm borrows, the real interest rate is a direct cost of investment. Each dollar borrowed to finance purchasing new capital incurs an explicit interest cost to the firm.

b. The opportunity cost of using the retained earnings for investment is that they can be used to purchase a bond earning interest. Thus, the real interest rate is an implicit opportunity cost of investment if the firm uses internal finance.

c. Tobin's q depends on share prices, which are the present value of expected future profits of the firm. A higher interest rate increases the discount rate applied to these future earnings and thus lowers their present value. An increase in the interest rate should lower stock prices and q, lowering the attractiveness of investment if financed by issuing new shares.

6. Using U.S. aggregate annual data from 1979 to 2001, the correlation between real investment and the real corporate-bond interest rate is –0.35, whereas the correlation between real investment and real GDP is 0.96. How, if at all, does this information help you assess the relative merits of the various investment theories we have considered. Can you be confident of the direction of causality between these pairs of variables? What difficulties does ambiguity of causality present in understanding and testing investment theory?

The high positive correlation between investment and income/output could be evidence in support of the accelerator theory, or it could be reverse causality showing that autonomous changes in investment affect output through the multiplier effect. Clearly, the correlation is strong, which suggests that one or both of these channels may be important. The correlation between investment and interest rates is negative, as theory suggests, but quite small. This could happen for several reasons, including the presence of lags in the investment relationship, difficulties in measuring the real interest rate, and the relative lack of variability in ex-ante real interest rates.