

Using Indifference Curves to Analyze Intertemporal Substitution

We are most comfortable using indifference curves to analyze consumption choices, and this tool can easily be used to explain the intertemporal substitution model and consumption smoothing. Of course, relying on indifference curves allows us to examine only two dimensions at a time, so we can apply this method only to the two-period model.

Suppose that utility is given by

$$U = u(C_1) + e^{-\rho}u(C_2).$$

Even though the two-period model requires that we work in discrete time, we shall use continuously compounded discounting to retain more symmetry with the Ramsey framework. With continuous compounding of interest, the individual's budget constraint is

$$W_1 + e^{-r}W_2 \equiv Y = C_1 + e^{-r}C_2.$$

From the consumer's standpoint, Y and r are given, ρ is a parameter of the utility function, and C_1 and C_2 are the individual's constrained choices.

Consider first the consumer's budget constraint. We will plot C_1 on the horizontal axis and C_2 on the vertical axis, so we begin by solving for C_2 to get $C_2 = Y/e^{-r} - C_1/e^{-r} = e^r Y - e^r C_1$. This is a straight line intersecting the vertical axis at $e^r Y$ (and the horizontal axis at Y) and having a slope equal to $-e^r$.

Next consider the consumer's indifference curves. These will not be linear; their exact form depends on the functional form of the function $u(\cdot)$. All we know about u are the assumptions that $u' > 0$ and $u'' < 0$. An indifference curve corresponding to a given level of utility U_0 is defined as the set of (C_1, C_2) for which $U = U_0 = u(C_1) + e^{-\rho}u(C_2)$. We are interested in the slope of the indifference curve, which is the change in C_2 that leaves utility unchanged ($dU = 0$) following a unit change in C_1 :

$$\left. \frac{dC_2}{dC_1} \right|_{dU=0}.$$

Since we can't solve for C_2 as an explicit function of C_1 and U_0 , we can only obtain the slope of the indifference curve by implicit differentiation. We begin by taking the total differential of the utility function:

$$dU = u'(C_1)dC_1 + e^{-\rho}u'(C_2)dC_2.$$

Utility is not changing along an indifference curve, so we set $dU = 0$ and solve for dC_2/dC_1 :

$$\left. \frac{dC_2}{dC_1} \right|_{dU=0} = -\frac{u'(C_1)}{e^{-\rho}u'(C_2)} = -e^{\rho} \frac{u'(C_1)}{u'(C_2)}.$$

As we move from lower right to upper left in the positive quadrant, C_1 gets smaller and C_2 gets larger. The negative second derivative of the utility function assures us that $u'(C_1)$ gets larger as C_1 gets smaller and $u'(C_2)$ gets smaller as C_2 gets larger, so $u'(C_1)/u'(C_2)$ increases as we move from lower right to upper left: the indifference curves get steeper and are convex in the usual way.

We know that (barring a corner solution, which is improbable here) the consumer maximized utility by consuming at a point where an indifference curve is tangent to her budget constraint. This tangency occurs at a point where the slope of the indifference curve equals the slope of the budget constraint. Recall that the slope of the budget constraint is $-\ell'$, so the mathematical equilibrium condition is $-\ell^p u'(C_1)/u'(C_2) = -\ell'$, or $u'(C_1)/u'(C_2) = \ell^{(r-p)}$.

Consider first the case where the rate of return on capital r is exactly equal to the consumer's marginal rate of time preference ρ . In this case $-\ell^p = -\ell'$, so the tangency condition becomes $u'(C_1)/u'(C_2) = 1$, or $u'(C_1) = u'(C_2)$. Because $u'' < 0$, $u'(C_1) = u'(C_2)$ if and only if $C_1 = C_2$, which means in geometric terms that the tangency between the indifference curve and the budget line must line on the 45° line through the origin. In economic terms, with the interest rate equals the rate of time preference, an individual will choose equal consumption in both periods: she will *smooth* consumption. Figure 1 shows this consumer equilibrium situation at point a .

Intuitively, the interest rate is the reward to saving offered by the market and the rate of time preference is the reward demanded by the consumer to justify postponing consumption. If the two are exactly in balance, then the consumer chooses future consumption equal to present consumption. If, further, the consumer's income in the two periods is equal, she will choose zero saving. If she has higher income in one period than the other, she will save in the higher-income period and dissave in the lower-income period to smooth her consumption.

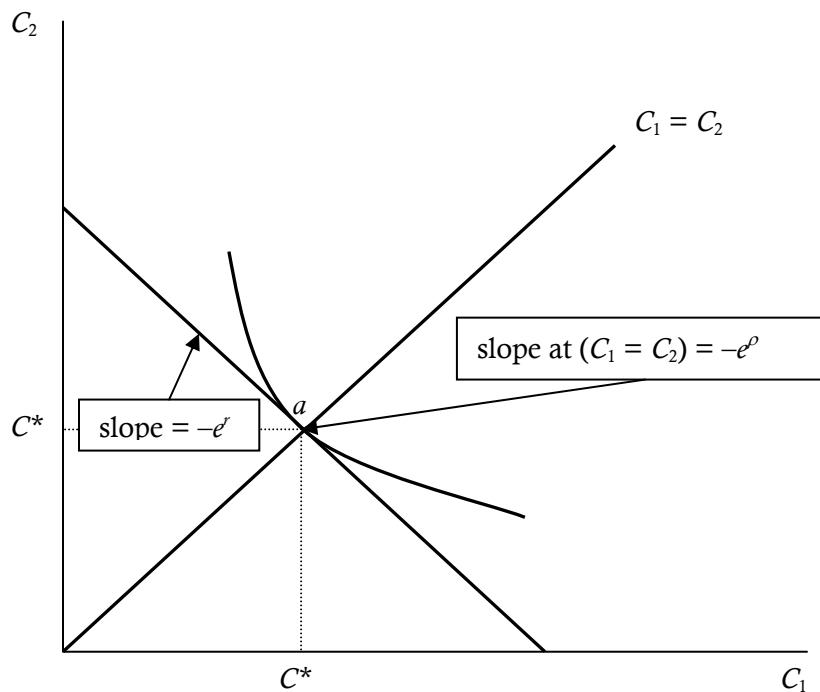


Figure 1. Consumer equilibrium when $r = \rho$

We have established the perfect consumption-smoothing outcome when $r = \rho$. What happens when $r > \rho$ or $r < \rho$? Once again, we can answer these questions easily with the indifference-curve diagram.

If $r > \rho$, then $e^r > e^\rho$ and the budget constraint is steeper than the indifference curve when $C_1 = C_2$. This means that the tangency must occur at a point on the budget constraint above and to the left of point a in Figure 1, where $C_1 < C_2$. In terms of the mathematical equilibrium condition, $e^{(r-\rho)} > 1$, so $u'(C_1)/u'(C_2)$ must be greater than one and $u'(C_1) > u'(C_2)$. With marginal utility decreasing in consumption, $u'(C_1) > u'(C_2)$ if and only if $C_2 > C_1$. Intuitively, when $r > \rho$, the consumer chooses more consumption in the future than in the present. She wants a consumption path that rises over time because the market reward to saving (r) exceeds her innate desire to consume early (ρ). If her income is the same in both periods, she will choose positive saving in period one. (However, it is important to note that if her income in period two is considerably higher than period one she may dissave in period one, although her consumption in two will still be higher than in one.)

The exact opposite happens if $r < \rho$. The budget constraint is flatter than the indifference curves at $C_1 = C_2$, so the tangency occurs below and to the right of Figure 1's point a , where $C_1 > C_2$. Mathematically, $e^{(r-\rho)} < 1$, so at equilibrium $u'(C_1) > u'(C_2)$, which implies $C_2 < C_1$. In terms of intuition, when $r < \rho$, the market reward to postponing consumption falls short of the consumers innate desire for current consumption, thus she chooses higher consumption now and lower consumption in the future. If her income is equal in both periods, she will dissave in the current period to finance high current consumption at the expense of lower future consumption when she must pay off her debt in the future. (Once again, if her income were enough lower in the future period, she might actually choose positive saving in period one, but she will not save enough to smooth her consumption perfectly.)