
Partner assignments

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Each team should submit a single paper. I strongly encourage you to work as a team in solving the problems. At a very minimum, both partners must understand everything that's on the paper when you turn it in. You are free to interact with other teams and to seek advice from me as you work on the problems, especially on the mathematical solutions. The idea of problem sets is to learn from them, not to test whether you can do them perfectly without help.

Problems

1. Marginal products in terms of intensive production function. In the Solow growth model, we write the aggregate production function as $Y = F(K, AL)$ and the intensive production function as $y = f(k)$, where $y \equiv \frac{Y}{AL}$, $k \equiv \frac{K}{AL}$, and $f(k) \equiv F\left(\frac{K}{AL}, 1\right)$. Assume constant returns to scale and the usual properties for the partial derivatives of F .

- Show that the marginal products of capital and labor are $\frac{\partial Y}{\partial K} = f'(k)$ and $\frac{\partial Y}{\partial L} = A[f(k) - kf'(k)]$.
- Show that the marginal product of capital is decreasing in k and the marginal product of labor is increasing in k .
- In competitive equilibrium, each unit of capital and labor will be paid its marginal product. Based on part a, how do the real rental rate of a unit of capital ($r + \delta = \text{MPK}$) and the real wage on labor ($w = \text{MPL}$) behave on a steady-state growth path? Does this seem realistic given the long-run behavior of the U.S. economy?
- How do total earnings of capital ($K \times \text{MPK}$) and total earnings of labor ($L \times \text{MPL}$) behave along the growth path? Given the behavior of total output, what will happen to capital's share (capital earnings divided by total output) and labor's share in steady-state growth?

2. Existence of aggregate production function. Suppose that the economy consists of n firms indexed by $i = 1, 2, \dots, n$. Each firm operates in perfectly competitive markets for output, labor, and capital, with equilibrium nominal prices P , W , and rP , respectively, that are common to all firms. Let $w = W/P$ be the real wage and r be the real price of capital services. (Assume no depreciation for this problem.) The i th firm produces output Y_i using inputs K_i and L_i according to a common production function

$$Y_i = F(K_i, L_i),$$

with constant returns to scale and the usual marginal properties:

$$F_{K_i} > 0, \quad F_{L_i} > 0, \quad F_{K_i K_i} < 0, \quad F_{L_i L_i} < 0.$$

The real profits of firm i (expressed in units of output) are

$$\Pi_i = Y_i - wL_i - rK_i = F(K_i, L_i) - wL_i - rK_i.$$

- Suppose that firm i maximizes its profit by choosing K_i and L_i given the prevailing relative prices. Find the first-order conditions for profit maximization by solving $\frac{\partial \Pi_i}{\partial K_i} = \frac{\partial \Pi_i}{\partial L_i} = 0$.
- Use the first-order conditions to show that all firms have common equilibrium values for the marginal products of labor and capital and the capital/labor ratio, provided that they all have a common production function and face common prices.
- Euler's theorem on homogeneous functions tells us that, if F has constant returns to scale, $Y_i = F_{K_i}(K_i, L_i) \cdot K_i + F_{L_i}(K_i, L_i) \cdot L_i$. Use this result to show that under our assumptions an aggregate production function exists. In words, show that total output depends only on aggregate inputs, *i.e.*, $Y = F(K, L)$, where $Y \equiv \sum_{i=1}^n Y_i$, $K \equiv \sum_{i=1}^n K_i$, $L \equiv \sum_{i=1}^n L_i$. Depending on your strategy of proof, it may be useful to recall from the previous problem that the marginal products depend only on the capital/labor ratio.
- Explain intuitively why no aggregate production function would exist under increasing or decreasing returns to scale.

3. Work Romer's problem 1.3.

- **Addition to each part:** "Sketch the new steady-state growth path of Y and the path that Y will follow as it converges to its new steady-state path. Explain in words what is happening at the moment of the change and afterwards."
- Be careful on part (c): do you know if $k < 1$ or $k > 1$?

4. Work Romer's problem 1.4.

- Assume that the new workers bring no capital with them and that their arrival has no effect on A .

- **Addition** to part (a): “What happens to total output Y at the time of the jump? Why?”
- **Addition** to part (c): “Is the total level of output on the new balanced growth path higher, lower, or the same as it would have been if there had been no new workers? Why?”