Economics 354 **Spring 2008**
Homework Solutions

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1. Consider the model whose game tree is depicted in Scotchmer's Figure 5.4. This model assumes that Firm 1 chooses to invest in the basic research that is needed for the application to be discovered and that Firm 2's investment in the application is contingent of being able to earn positive profits from the project. Innovation pairs are characterized by the parameters (x, y, c_1, c_2). Other variables in the model are the profit share under patent protection π, the discounted patent length T, and the deadweight loss share ℓ.

Suppose that $T = 10$, $\pi = 0.5$, $\ell = 0.25$, and $r = 0.05$. Consider the four innovation pairs I, II, III, and IV with parameters shown in the table below:

Innovation pair	x	y	c_1	c_2
I	500	1000	2000	2000
II	0	1000	2000	2000
III	0	1000	2000	2700
IV	0	1000	2500	2700

For each of these pairs, calculate:

i. the total net social benefit from the innovation pair if both innovations are put in the public domain,

$$\text{Net social benefit} = \frac{1}{r}(x + y) - c_1 - c_2$$

ii. the total benefit to users if the two innovations are undertaken under patent protection,

$$\text{User benefit under patent} = \left(\frac{1}{r} - (\pi + \ell)T\right)(x + y)$$

iii. the total net benefit to a combined firm undertaking both innovations under patent protection,

$$\text{Net benefit to joint firm} = \pi T(x + y) - c_1 - c_2$$

iv. the threat points of each firm under (1) ex-post licensing and (2) ex-ante licensing,

$$\text{Firm 1 ex-post and ex-ante threat point} = \pi T x - c_1$$
$$\text{Firm 2 ex-post threat point} = -c_2$$
$$\text{Firm 2 ex-ante threat point} = \begin{cases} -c_2 \text{ if } \tfrac{1}{2} y\pi T > c_2, \\ 0 \text{ otherwise.} \end{cases}$$

v. the total gains to the two firms from (1) ex-post licensing and (2) ex-ante licensing,

$$\text{Total gains ex-post} = \pi T y,$$
$$\text{Total gains ex-ante} = \pi T y - c_2$$

vi. the payoffs to each firm under ex-post licensing assuming Nash bargaining: that the total gains from licensing are divided equally,

$$\text{Ex-post payoff for 1} = \pi T\left(x + \tfrac{1}{2} y\right) - c_1$$
$$\text{Ex-post payoff for 2} = \tfrac{1}{2}\pi T y - c_2$$

vii. the payoffs to each firm under ex-ante licensing assuming Nash bargaining.

$$\text{Ex-ante payoff for 1} = \begin{cases} \pi T\left(x + \tfrac{1}{2} y\right) - c_1, \text{ if } \tfrac{1}{2}\pi T y > c_2 \\ \pi T\left(x + \tfrac{1}{2} y\right) - c_1 - \tfrac{1}{2} c_2, \text{ otherwise} \end{cases}$$
$$\text{Ex-ante payoff for 2} = \begin{cases} \tfrac{1}{2}\pi T y - c_2, \text{ if } \tfrac{1}{2}\pi T y > c_2 \\ \tfrac{1}{2}\left(\pi T y - c_2\right), \text{ otherwise} \end{cases}$$

Based on these calculations,

a. Would this innovation pair be socially beneficial if a government performed both innovations (using tax money raised through lump-sum taxes) and offered the resulting products free?
b. Would this innovation pair be profitable under the current patent regime if both innovations were done jointly by the same firm?
c. Would it be feasible for this innovation pair to emerge through ex-post licensing between two firms?
d. Would it be feasible for this pair to emerge through ex-ante licensing between two firms?

What outcome do you predict in each case? Who gains and loses the most in each case relative to the ideal outcome?

Calculations in table below:

Innovation pair:	I	II	III	IV
Net social benefit	26,000	16,000	15,300	14,800
User benefit under patent	18,750	12,500	12,500	12,500
Net benefit to joint firm	3,500	1,000	300	–200
Firm 1 threat point	500	–2,000	–2,000	–2,500
Firm 2 ex-post threat point	–2,000	–2,000	–2,700	–2,700
Firm 2 ex-ante threat point	–2,000	–2,000	0	0
Total gains ex-post	5,000	5,000	5,000	5,000
Total gains ex-ante	3,000	3,000	2,300	2,300
Firm 1 ex-post payoff	3,000	500	500	0
Firm 2 ex-post payoff	500	500	–200	–200
Firm 1 ex-ante payoff	3,000	500	–850	–1,350
Firm 2 ex-ante payoff	500	500	1,150	1,150

Case I: It is social beneficial both through public domain and joint-firm innovation. Ex-post licensing is feasible and that will be the outcome. Although Firm 2 would do better if it could get Firm 1 to share costs, Firm 1 will not be willing to do any worse that it would do with ex-post bargaining, hence Firm 2 cannot extract any better deal ex-ante than it could get ex-post. Consumers lose quite a bit here relative to the public-domain solution (if that were feasible). Firm 2 doesn't gain much out of its valuable innovation, but then it couldn't have been done without Firm 1, which gets most of the benefit.

Case II: Similar general outcome to Case I, but Firm 1's profit position is weaker here. In both cases it gets all the direct gains from its own innovation and some of the gains from Firm 2's, but here there are no gains from its own innovation. Each firm gets an equal payoff here.

Case III: A more interesting case. The innovation pair is socially beneficial. Firm 2 won't do ex-post licensing here and Firm 1 knows it. The fact that Firm 1 has (by assumption) already made its discovery but Firm 2 has not yet done its R&D puts the latter at an advantage. Firm 1 has no bargaining power here because if there is no bargain at all it is out the entire 2,000 it spent on R&D and gets no revenue without the follow-up innovation. Firm 1 clearly loses out in this case. If it had known what it was getting into, it would not have done the initial innovation without an "ex-ante ex-ante" agreement to share the collective gains (300). However, the Scotchmer model assumes that Firm 1 has already invested.

Case IV: Another interesting case. The innovations are clearly socially beneficial, but with the prevailing discounted patent length and profit rate, even the two firms working together would not find it profitable to undertake the pair. In this case, there is no licensing scheme that is profitable to Firm 1, so it should not do R&D. However, if Firm 1 has already foolishly bumbled into its innovation, it can recoup some of its sunk cost through ex-post licensing with Firm 2.

2. As noted in the previous problem, Scotchmer's Figure 5.4 model assumes that the basic research has already been done by Firm 1. The difference between ex-ante and ex-post licensing is whether it is done before or after Firm 2 invests. What would be the outcome of before-basic-research licensing if the two firms were to bargain before either had invested? Is there ever a case where this outcome would be better than either after-basic-research bargaining scheme can achieve? Explain.

Discussed above with respect to III and IV.

3. Explain why in the model of Scotchmer's Figure 4.1, intellectual property (even with infinite patent length) can never achieve the socially optimal level of innovation. Would this answer be different if innovators could be perfect price discriminators in the market for their innovated products? Would you advocate such a policy? Why or why not?

Without price discrimination, there will always be deadweight loss, which means that the firm's return from R&D will always be less than the social gain even if patents are eternal. Under perfect price discrimination, firms could capture all gains and there would be no deadweight loss, but also no social benefit to anyone except the firm! As long as there are social benefits external to the firm, there will be some socially desirable projects that are not undertaken.

4. Suppose that in the model of Scotchmer's Figure 4.2 there is a project in which each firm that does R&D has a 5% chance of success per year of R&D at a cost of $200,000 per year. Discounted patent length is 18 years, the profit rate π is 0.50, deadweight loss percentage ℓ is 0.25, and the interest rate is 5%. The social value of the innovation is $500,000 per year.

For all values of n between zero and 15, calculate the total expected social benefit and total social cost, net expected social benefit (total expected social benefit minus total social cost), and expected net gain for the marginal entrant. (You may find it convenient to use Excel formulas to do this.) What would be the socially optimal number of firms doing R&D? How many would enter? Is discounted patent length too short or too long in this case? What would be the optimal discounted patent length? How much additional social benefit is gained by having the optimal length rather than 18 years?

The total, gross social gain if the R&D is successful is $S = v\left(\frac{1}{r} - \ell T\right) = 7{,}750{,}000$. The private gain is $\Pi = \pi v T = 4{,}500{,}000$. The table below summarizes the relevant numbers for n from 1 to 15. Five firms would enter; the expected private gain of the sixth would be negative. The socially optimal number of entrants is 13, so the reward to R&D is too low and discounted patent length is too short.

Number of entrants	Probability of success	Gross exp. social gain	Gross exp. private gain	Total R&D costs	Net exp. social gain	Net exp. private gain from entry	Net exp. marginal social gain
n	$P(n)$	$P(n)S$	$P(n)\Pi$	cn	$P(n)S - cn$	$P(n)\Pi/n$	$[P(n) - P(n-1)]S - c$
1	0.050	387,500	225,000	200,000	187,500	25,000	187,500
2	0.098	755,625	438,750	400,000	355,625	19,375	168,125
3	0.143	1,105,344	641,813	600,000	505,344	13,938	149,719
4	0.185	1,437,577	834,722	800,000	637,577	8,680	132,233
5	0.226	1,753,198	1,017,986	1,000,000	753,198	3,597	115,621
6	0.265	2,053,038	1,192,086	1,200,000	853,038	-1,319	99,840
7	0.302	2,337,886	1,357,482	1,400,000	937,886	-6,074	84,848
8	0.337	2,608,492	1,514,608	1,600,000	1,008,492	-10,674	70,606
9	0.370	2,865,567	1,663,878	1,800,000	1,065,567	-15,125	57,075
10	0.401	3,109,789	1,805,684	2,000,000	1,109,789	-19,432	44,222
11	0.431	3,341,799	1,940,400	2,200,000	1,141,799	-23,600	32,011
12	0.460	3,562,209	2,068,380	2,400,000	1,162,209	-27,635	20,410
13	0.487	3,771,599	2,189,961	2,600,000	1,171,599	-31,541	9,390
14	0.512	3,970,519	2,305,463	2,800,000	1,170,519	-35,324	-1,080
15	0.537	4,159,493	2,415,189	3,000,000	1,159,493	-38,987	-11,026

Repeating the experiment for different patent lengths leads to the conclusion that a patent length of 21 results in 12 entrants, which is the socially optimal number when patents are this long. Total expected social gains with 12 entrants at patent length 21 is 989,844, which exceeds the achieved level of 753,198 that occurs with $T = 18$ and $n = 5$.

It is also interesting to think about expected gains to users of the innovation, which is the difference between gross social gains and gross private gains: $P(n)\,(S - \Pi)$. Longer patents induce more R&D and thus a higher probability of success, but also capture a larger share for the successful firm, so it is not clear whether users are better off or whether all of the social gains are going to the firms. Under the shorter patent length with five firms entering, user gains are 735,212; with the longer patents and more innovation, users get 976,735. So users of the innovation are better served by the longer patent length and greater probability of success.