

Laboratory I: Impedance

Reading: Simpson Chapter 1 (if necessary) & Chapter 2 (particularly 2.9-2.13)

1 Introduction

In this first lab we review the properties of the three basic passive circuit elements: the resistor, the capacitor, and the inductor. We will need a quick review of the theory behind AC-circuits and how the concept of a complex impedance can be used to significantly simplify the analysis of such circuits. The experimental challenge in this lab will be to determine unknown impedance using the existing measurement equipment. I want you to have the following setup in mind.

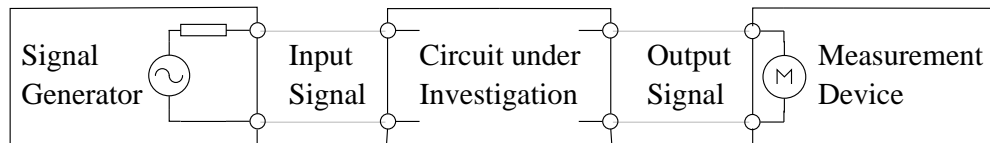


Figure 1: General experimental setup. Note that the signal generator and the measurement device, e.g. an oscilloscope, are part of the “overall circuit”. One needs to know the electrical properties of these devices in order to make correct measurements.

2 Linear Passive Circuit Elements

First let me say a few words about what we mean when we say circuit elements are *passive* and *linear*.

2.1 Passive Circuit Elements dissipate Power

Resistors, capacitors and inductors are *passive* circuit elements because they contain no internal sources of energy. That is, when a signal is applied to passive circuit elements the electrical power of the signal either decreases or stays constant. In contrast, active circuit elements, such as operational amplifiers, can increase the power of a signal and must be supplied with both an input signal and a source of power (typically a battery or DC power supply).

Let us digress here shortly and remind ourselves of the definition of power for DC- and AC-circuits.

For DC-circuits, where both the current and voltage do not change in time, the power P is unambiguously defined as the product of current and voltage.

$$P = IV \quad (1)$$

Thus, passive circuit elements may increase the voltage but then the current has to decrease accordingly such that the product IV of the signal before the passive circuit element is larger or equal to that after the passive circuit element.

The same holds for AC-circuits. However, thinking about power flow in AC-circuits, where charges “slosh” back and forth, is slightly less straightforward and one has to distinguish between average and instantaneous power. To be more precise, when analyzing AC-circuits we will be thinking of simple sinusoidal signals, which we can represent mathematically as

$$i(t) = I_0 \cos(\omega t + \varphi_I) \quad (2)$$

$$v(t) = V_0 \cos(\omega t + \varphi_V). \quad (3)$$

In other words, the voltage that we measure varies sinusoidally and is fully characterized by the fixed values for its real amplitude V_0 , angular frequency $\omega = 2\pi f$ (f =frequency), and constant phase φ_V . The same holds for the current, which will have an amplitude I_0 and a constant phase φ_I that might in general be different from φ_V . On the other hand, the frequency f with which the current oscillates is the same as that of the corresponding voltage. The instantaneous power $p(t)$ is then simply defined as the product of the sinusoidally varying current and voltage and therefore varies periodically in time.

$$p(t) = i(t)v(t) \quad (4)$$

A more useful quantity is the average power of such sinusoidal signals, which is defined as the instantaneous power averaged over one oscillation period.

$$P_{av} = \frac{1}{T} \int_0^T p(t) dt \quad (5)$$

$$= \frac{1}{T} \int_0^T [I_0 \cos(\omega t + \varphi_I)] [V_0 \cos(\omega t + \varphi_V)] dt \quad (6)$$

$$= \frac{I_0 V_0}{2T} \int_0^T [\cos(\varphi_I - \varphi_V) + \cos(2\omega t + \varphi_I + \varphi_V)] dt \quad (7)$$

$$= \frac{I_0 V_0}{2} \cos(\varphi_I - \varphi_V) \quad (8)$$

$$= I_{rms} V_{rms} \cos(\varphi_I - \varphi_V) \quad (9)$$

Note that the average power depends on the phase difference.

For resistors the voltage and current are in-phase ($\varphi_V = \varphi_I$), which means that the resistor dissipates power: $P_{av} = I_{rms} V_{rms}$. As a result, resistive elements generate heat and can get very hot if the current through them becomes large. In contrast, if φ_V and φ_I are 90° out of phase the \cos -term is zero and no power is dissipated. This is the case for ideal inductors and capacitors (The actual devices that you find in the lab do of course have a small resistance).

To summarize, passive elements such as resistors, capacitors, and inductors will decrease the average power of signals in an AC-circuit and for idealized circuit elements conservation of signal power is possible.

2.2 Linear Circuit Elements

In addition to being passive circuit elements, resistors, capacitors, and inductors are *linear* circuit elements because the relation between the current through them and the voltage across them is linear, at least ideally:

$$\text{Resistor} \quad v(t) = R i(t) = R \dot{Q} \quad (10)$$

$$\text{Inductor} \quad v(t) = L \frac{d}{dt} i(t) = L \ddot{Q} \quad (11)$$

$$\text{Capacitor} \quad v(t) = \frac{1}{C} \int i(t) dt = \frac{1}{C} Q \quad (12)$$

Linearity is most easily recognized for the resistor, where a plot of the voltage across a resistor plotted against the current flowing through that resistor (I/V curve) results in a straight line with slope $1/R$. Note that the voltage across an inductor is related to the *derivative* of the current and the voltage across a capacitor is related to the *integral* of the current. To see that these relations are nevertheless linear, we need to recall that a mathematical operator (here denoted by \mathcal{L}) is said to be linear if the following holds:

$$\mathcal{L}[av_1(t) + bv_2(t)] = a\mathcal{L}[v_1(t)] + b\mathcal{L}[v_2(t)] \quad (13)$$

where a and b are constants and v_1 and v_2 are (in this context) functions of time. For example, the derivative is a linear operator because, if we set $\mathcal{L}[\cdot] = \frac{d}{dt}[\cdot]$, the simple relation

$$\frac{d}{dt}[av_1(t) + bv_2(t)] = a \frac{dv_1(t)}{dt} + b \frac{dv_2(t)}{dt} \quad (14)$$

implies linearity. Integration is also a linear operation because we may set $\mathcal{L}[\cdot] = \int[\cdot]dt$ and the relation

$$\int [av_1(t) + bv_2(t)] dt = a \int v_1(t) dt + b \int v_2(t) dt \quad (15)$$

holds.

As we just showed, inductor and capacitor are both linear devices because differentiation and integration are both linear operations. Linearity is significant because it permits us to predict mathematically what the voltage and current will be at all points in the circuit at all times. Nonlinear elements like diodes can make the analysis of a circuit very hard indeed, such a circuit can even behave chaotically!

One of the most important consequences of dealing with circuits made from “passive linear” elements is that we know that if our input signal oscillates with frequency f then the output signal will also oscillate with the same frequency f . This is not true for circuits including “nonlinear” elements such as diodes. Nonlinear circuits may transfer some signal power from the input frequency f to other frequencies such as $2f$ in the output signal.

The fact that the measured signals remain at the same frequency in linear circuits allows one to greatly simplify the analysis. All we need is to figure out how the amplitudes and phases of the signals are related. This task is most easily achieved using complex variables or the concept of *complex impedance*.

3 Complex impedance

Resistors are pretty straightforward – using Kirchhoff's Laws we can easily analyze a circuit with only resistors in it to find the voltage at every point and the current through every resistor. *Kirchhoff's laws* state that

- (1) the sum of all the voltage changes as you follow around a loop in a circuit is always exactly zero, and
- (2) the current flowing into any point in a circuit is equal to the current flowing out of it.

Kirchhoff's laws apply to all circuits, so there is still a straightforward way to handle circuits containing capacitors and inductors. However, equations for circuits with capacitors and inductors are trickier because the current does not appear all by itself, but rather in an integral or derivative because of Eq.(11) and Eq.(12).

Fortunately, there is a clever way to use complex variables to avoid solving any differential equations for the case of *linear* circuits. All we need to know are the rules for manipulating complex numbers and variables, plus a few extra rules in applying them to linear circuits.

3.1 The briefest review of complex variables

1. There are two ways to represent a complex variable:
 - (i) as the sum $x + iy$, where x is the real part, y is the imaginary part, and $i = \sqrt{-1}$;
 - (ii) as the product $re^{i\phi}$, where r is the amplitude, and ϕ is the phase.

Switching between the two notations is not too hard:

$$x = r \cos(\phi) \qquad r = \sqrt{x^2 + y^2} \qquad (16)$$

$$y = r \sin(\phi) \qquad \phi = \tan^{-1}(y/x) \qquad (17)$$

You need to be comfortable with both representations.

2. The complex conjugate of a complex number is formed by changing the sign of all of the imaginary terms in the complex number. For example, if $z = (a + ib)/(c - id)$, then the complex conjugate $z^* = (a - ib)(c + id)$. Notice that the amplitude (also known as the magnitude) is given by

$$r = |z| = \sqrt{x^2 + y^2} = \sqrt{(x + iy)(x - iy)} = \sqrt{zz^*}. \qquad (18)$$

3. The real part/imaginary part notation is most useful when you are going to add or subtract. The amplitude/phase notation is best for multiplication and division. Add (subtract) two complex numbers by separately adding (subtracting) their real parts and their imaginary parts. Multiply (divide) two complex numbers by multiplying

(dividing) their amplitudes and adding (subtracting) their phases. Letting $z_n = x_n + iy_n = r_n e^{i\phi_n}$, the rules are:

$$\text{if } z_3 = z_2 + z_1, \quad \text{then } z_3 = (x_2 + x_1) + i(y_2 + y_1) \quad (19)$$

$$\text{if } z_3 = z_2 - z_1, \quad \text{then } z_3 = (x_2 - x_1) + i(y_2 - y_1) \quad (20)$$

$$\text{if } z_3 = z_2 z_1, \quad \text{then } z_3 = r_2 r_1 e^{i(\phi_2 + \phi_1)} \quad (21)$$

$$\text{if } z_3 = z_2/z_1, \quad \text{then } z_3 = (r_2/r_1) e^{i(\phi_2 - \phi_1)} \quad (22)$$

Note that you don't *have* to do multiplication and division using the amplitude/phase representation, it just makes the algebra simpler. You could also write

$$\text{if } z_3 = z_2 z_1, \quad \text{then } z_3 = (x_2 x_1 - y_2 y_1) + i(x_2 y_1 + y_2 x_1) \quad (23)$$

and

$$\begin{aligned} \text{if } z_3 = z_2/z_1, \quad \text{then } z_3 &= \frac{x_2 + iy_2}{x_1 + iy_1} \\ &= \frac{x_2 + iy_2}{x_1 + iy_1} \frac{x_1 - iy_1}{x_1 - iy_1} \\ &= \frac{(x_2 x_1 + y_2 y_1) + i(x_1 y_2 - y_1 x_2)}{x_1^2 + y_1^2}. \end{aligned} \quad (24)$$

3.2 Why complex notation

For resistors in both DC- and AC-circuits $V = RI$ holds, which makes analysis simple. By letting V and I be complex variables one can define a generalized resistance or *complex impedance* Z for capacitors and inductors, such that

$$V = ZI \quad (25)$$

holds. With this, it is then possible to use the same techniques that are familiar from analyzing resistor networks to analyze arbitrary networks of passive linear circuit elements.

To illustrate, consider a voltage $v = V_0 \cos(\omega t) = \text{Re}(V_0 e^{i\omega t})$ that is applied to an inductor. We would like to know the corresponding current. One way to solve this problem is to use Eq.(11). To satisfy Eq.(11), we can integrate and find that the corresponding current has to be

$$i(t) = \frac{1}{L} \int v(t) dt = \frac{1}{L} \int V_0 \cos(\omega t) dt = \frac{V_0}{\omega L} \sin(\omega t) = \frac{V_0}{\omega L} \cos(\omega t - \pi/2).$$

Converting to complex notation yields

$$i(t) = \text{Re} \left(\left[\frac{V_0}{\omega L} e^{-i\pi/2} \right] e^{i\omega t} \right) = \text{Re} \left(\left[\frac{V_0}{i\omega L} \right] e^{i\omega t} \right).$$

We see that the real amplitude is given by $I_0 = \frac{V_0}{\omega L}$ and most importantly, that there is a constant phase-shift $\varphi_I = -\pi/2$. The current lags by 90° or $\pi/2$ rad behind the voltage. To find this result we had to solve the differential equation through integration, which is a lot of work.

Using complex notation we can find the amplitude and phase in a simpler manner. Write the voltage as $v = V_0 e^{i\omega t}$ (but of course what is measured is only the real part this), and define the complex impedance of the inductor as $Z = i\omega L$. Then use $V = ZI$ to find

$$i = \frac{v}{Z} = \left[\frac{V_0}{i\omega L} \right] e^{i\omega t} = \left[\frac{V_0}{\omega L} e^{-i\pi/2} \right] e^{i\omega t},$$

in agreement with the previous result. The main point is that all we ever need to figure out is the real amplitude and the constant phase – here $I_0 = V_0/(\omega L)$ and $\varphi_I = -\pi/2$. The “trick” was to define an appropriate complex impedance Z , where the complex-valued Z incorporates an appropriate factor relating the real-valued voltage-amplitude and current-amplitude and, in addition, Z incorporates the correct phase-shift between current and voltage.

It turns out that this technique of using complex impedance works in general. Here are the rules.

3.3 Rules for applying complex variables to circuits

1. *The impedances for resistors, inductors and capacitors are, respectively:*

$$\text{Resistor} \quad Z_R = R \quad (26)$$

$$\text{Inductor} \quad Z_L = i\omega L = \omega L e^{i\pi/2} \quad (27)$$

$$\text{Capacitor} \quad Z_C = \frac{-i}{\omega C} = \frac{1}{i\omega C} = \frac{1}{\omega C} e^{-i\pi/2} \quad (28)$$

I have given Z_C and Z_L in both the real/imaginary and amplitude/phase notations. Notice that reactive components (capacitors and inductors) have purely imaginary impedances and resistive components have only a real component.

Question: How does the impedance of each of these elements behave in the limits of very low and very high frequency?

Answer:

$$\begin{array}{ll} Z_R = R, & \text{for all } \omega \\ |Z_C| \rightarrow \infty, & \text{as } \omega \rightarrow 0 \\ |Z_L| \rightarrow 0, & \text{as } \omega \rightarrow 0 \end{array} \quad \begin{array}{ll} |Z_C| \rightarrow 0, & \text{as } \omega \rightarrow \infty \\ |Z_L| \rightarrow \infty, & \text{as } \omega \rightarrow \infty \end{array}$$

We should not be surprised to find that that Z_C and Z_L both depend on the frequency. Remember that a simple capacitor consists just of two conducting plates placed close to each other. In the very low-frequency limit ($\omega \rightarrow 0$) the plates act like an insulator

and pass no current, $I = 0$. Very high frequencies pass through a capacitor as if it were just a wire. Similarly, an inductor is just a coil of wire and therefore behaves like a wire at very low frequencies (since the wire has finite resistance, it actually behaves like a resistor at low frequencies). At high frequencies the difficulty of quickly varying the electromagnetic fields around the coil effectively chokes off the current through an inductor.

2. Write all sinusoidally varying voltages and currents as complex variables, $V_0 e^{i(\omega t + \varphi_V)}$ and $I_0 e^{i(\omega t + \varphi_I)}$.

Again, to be completely correct here, I should point out that the voltage and current are really just the real part of the complex variable, for example $V = \text{Re}(V_0 e^{i(\omega t + \varphi_V)}) = V_0 \cos(\omega t + \varphi_V)$. In practice, though, this never poses any problem: as long as we are using the amplitude/phase notation, taking the real part is completely trivial. Often only the amplitude is of interest, so if we are using the real part/imaginary part notation, the amplitude can be found readily using $|V| = \sqrt{VV^*}$.

Furthermore we can always pick one voltage (or current) in the circuit to have the constant phase equal to zero (e.g. $\varphi_V = 0$), though the voltages and currents at other points in the circuit may or may not have a constant phase equal to zero.

What if the voltage applied to the circuit is not sinusoidal? What if it is a square wave, for example? The answer is that for linear circuits we can always use Fourier analysis to decompose any signal into an infinite sum (or integral, for nonperiodic signals) of sinusoidal signals. Since in a linear circuit no power is transferred between the harmonic components, we can then analyze the circuit for each Fourier component separately. Again, the caveat is that this doesn't necessarily work for nonlinear circuits, and for these circuits it is often necessary to use numerical techniques.

3. For linear circuit elements, we have $V = IZ$, where V , I and Z are all complex variables.

Impedances add in series and in parallel just the way resistors do, but you must use the mathematics of complex variables throughout.

$$Z_{\text{series}} = Z_1 + Z_2 \qquad Z_{\text{parallel}} = Z_1 || Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Example: What is the total impedance $Z_R || Z_C$ of a resistor and capacitor in parallel?

Answer:

$$\begin{aligned} Z_R || Z_C &= \frac{Z_R Z_C}{Z_R + Z_C} = \frac{\frac{-iR}{\omega C}}{R - \frac{i}{\omega C}} = \frac{R}{i\omega RC + 1} \\ &= \frac{R}{i\omega RC + 1} \frac{-i\omega RC + 1}{-i\omega RC + 1} = \frac{R - i\omega R^2 C}{(\omega RC)^2 + 1} \end{aligned}$$

In the second line I have put the result in the standard form $x + iy$. As discussed next, this step is often not necessary.

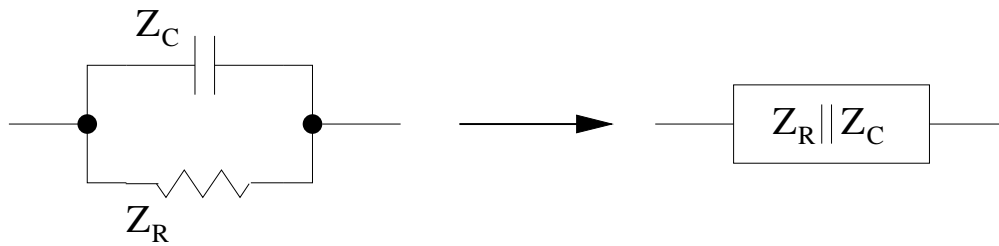


Figure 2: A resistor in parallel with a capacitor.

4. Very often it is only the magnitude of the impedance of a network that is of interest.

Often we do not measure the amount by which the phase is shifted as a sinusoidal signal passes through a network. In this case, we need only compute $|Z_{net}|$. How much work does this save us? The answer is, “Some, but not as much as you might like.” Basically, you can take the magnitude of a complex number without first putting the number in the standard form of $x + iy$ (which can make life easier), but you still must use complex arithmetic to combine all of the impedances to form Z_{net} before you can take the magnitude. Example:

- Longer way to compute $Z_R || Z_C$. Compute $Z_R || Z_C$ in standard $x + iy$ form as in the example above, then take the magnitude of the result. (Try this and see if you get the same result as the easy way.)
- Easier way to compute $Z_R || Z_C$

$$\begin{aligned}
 |Z_R || Z_C| &= \left| \frac{Z_R Z_C}{Z_R + Z_C} \right| = \frac{|Z_R Z_C|}{|Z_R + Z_C|} = \frac{\left| \frac{-iR}{\omega C} \right|}{\left| R - \frac{i}{\omega C} \right|} = \frac{\sqrt{\frac{-iR}{\omega C} \frac{iR}{\omega C}}}{\sqrt{(R - \frac{i}{\omega C})(R + \frac{i}{\omega C})}} \\
 &= \frac{\frac{R}{\omega C}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{R}{\sqrt{(\omega RC)^2 + 1}}
 \end{aligned}$$

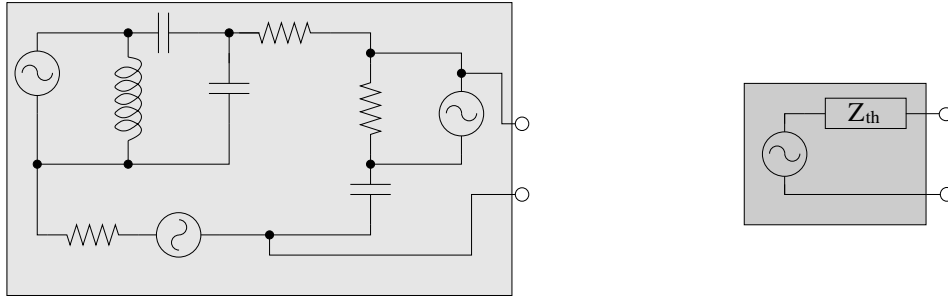
- Wrong way to compute $Z_R || Z_C$

$$|Z_R || Z_C| = \left| \frac{Z_R Z_C}{Z_R + Z_C} \right| = \frac{|Z_C| |Z_R|}{|Z_R| + |Z_C|} = \frac{\frac{R}{\omega C}}{R + \frac{1}{\omega C}} = \frac{R}{\omega RC + 1}$$

The above “wrong way” doesn’t work because the magnitude operation is not linear.

4 Thevenin's Theorem

In analyzing circuits it is often useful to simplify the circuit diagram. Thevenin's theorem is one way to achieve such simplification and it helps clarify our thinking.



Black Box

Thevenin Equivalent

Figure 3: Illustration of Thevenin's Theorem.

Picture a black box with a large number of different signal sources, resistors, capacitors and inductors inside, connected in any way whatsoever. Imagine that we have tapped into this network in exactly two places and these are connected to two terminals on the outside of the box. Thevenin's theorem states that any two-terminal network of signal sources, resistors, capacitors and inductors is equivalent to a single signal source in series with a single complex impedance. (You probably have encountered a simpler version of this theorem that applies only to DC voltage sources (batteries) and resistors.) Thus, for all we on the outside of the box can tell, there might as well be just a single signal source and a single complex impedance. This is a handy theorem, in part because it encourages us to think about circuits as being built up of a series of black boxes with relatively simple properties.

5 Input impedance, Output Impedance, Loading

Electronic engineers like to think of circuits as composed of “boxes” each of which has a certain function. Each box is often referred to as a “stage”, i.e. a “buffer stage” might be followed by a “filter stage” followed by an “amplifier stage”. Now when one hooks these “stages” together they can interact and problems may arise this way. In this context people often talk about the “load” that one stage puts on the previous one. This is an important issue, so let's discuss it with the help of an example.

Example: In Fig. 4 a), a function generator with a 50Ω output impedance forms the first stage (the one on the left). For concreteness, assume we set the generator controls such that it produces a sin-waveform. We now connect the generator's output terminals to

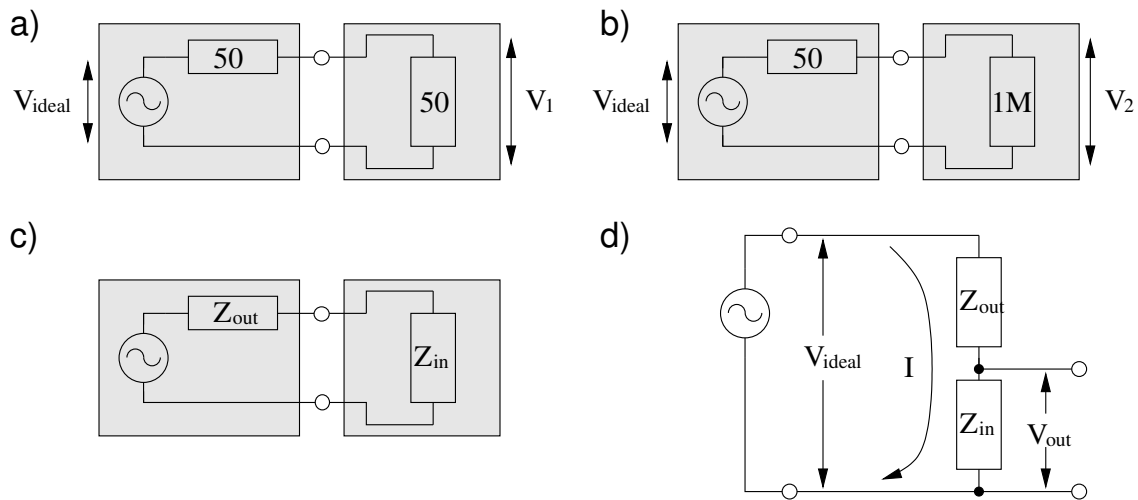


Figure 4: The input impedance of the second stage “loads down” the first stage. a) A $50\ \Omega$ function generator connected to a $50\ \Omega$ resistor. b) Connected to a $1\ \text{M}\Omega$ resistor. c) The general equivalent diagram for two connected stages. d) The redrawn circuit: An ideal voltage source followed by a voltage divider.

a second stage, a $50\ \Omega$ resistor, and measure across the resistor the voltage amplitude V_1 of the sin-wave. Then we repeat the experiment with a $1\ \text{M}\Omega$ resistor as shown in Fig. 4 b) and measure V_2 .

Naively we might expect that the voltage at the generator’s output terminals is always the same because we don’t touch the generator controls in between the experiments. Unfortunately, this is not the case because the two stages interact ($V_1 \neq V_2$). Let us work out what happens using a more general setting.

By Thevenin’s theorem, we can consider any circuit that we connect to as an ideal signal source in series with an impedance. Therefore, we can represent the first stage as shown in Fig. 4 c). By the same reasoning, the second stage, which we assume to be a black box circuit purely composed of passive circuit elements, can be considered as a single impedance. The equivalent impedance of the first stage is called its *output impedance*, Z_{out} . It is the impedance that one “sees” when “looking into” the output terminals of the first stage. The equivalent impedance of the second stage is called its *input impedance*, Z_{in} . It is the impedance that the first stage “sees” when “looking into” the input terminals of the second stage.

In this example, to figure out what the voltage is that the second stage sees, let us redraw the circuit as shown in Fig. 4 d). What we find is that this circuit looks like a voltage divider. You can now use what you have learned for voltage dividers consisting of two resistor and just replace R by Z . In this situation use

$$\begin{aligned} V_{out} &= I Z_{in} \\ I &= V_{ideal} / (Z_{out} + Z_{in}) \end{aligned}$$

to obtain that the voltage “seen” by the second stage

$$V_{out} = V_{ideal} \frac{Z_{in}}{Z_{out} + Z_{in}}. \quad (29)$$

Returning to the example of the function generator, we can now calculate V_1 and V_2 using Eq. (29):

$$V_1 = V_{ideal} \frac{50}{50 + 50} = \frac{V_{ideal}}{2}, \quad V_2 = V_{ideal} \frac{1 \cdot 10^6}{50 + 1 \cdot 10^6} \simeq V_{ideal}.$$

Thus, $V_2 = 2 V_1$, the voltage amplitude of the sin-wave that the function generator puts out changed by a factor of two because of the second stage!

As this example illustrates, if the equivalent impedance of the second stage is large compared to the equivalent impedance of the first stage, $|Z_{in}| \gg |Z_{out}|$, then the second stage will “see” a voltage approximately equal to V_{ideal} . Normally, this is exactly what we want because then we can think of the stages as independent – the second stage does not affect the first. However, if Z_{in} is not large compared to Z_{out} , then the second stage will “load down” the first. That is, the voltage entering the second stage, V_{out} , will no longer be even approximately equal to the ideal output voltage of the first stage, V_{ideal} . When one stage loads down another, there are two ways of looking at the problem: the output impedance of the first stage is too big or the input impedance of the second stage is too small.

The reason that you should be aware of this is that in the design of circuits the condition $|Z_{in}| \gg |Z_{out}|$ often means that you cannot just take the output of one circuit and connect it to the input of the next. Instead you have to build a buffer stage. Quite often a unity-gain voltage follower circuit will do (Fig. 5).

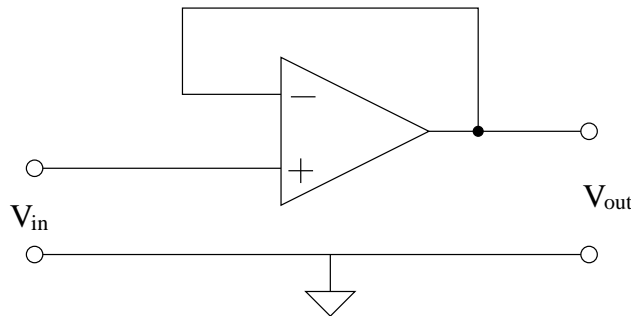


Figure 5: Unity-gain voltage follower circuit is constructed using an operational amplifier. The operational amplifier is an active circuit element with a very high input impedance and the ability to source quite a bit of current (low output impedance).

A second reason to be aware of the issue of loading is that on some instruments, such as high-end oscilloscopes, you have the option to switch the input impedance from a “high Z ” to a “50 Ω ” setting. If you choose $Z_{in} = 50\Omega$ you will almost surely load down the circuit

that you are measuring and will get unexpected results. (As an aside: the reason that a 50Ω option exists is that for radio-frequency circuits the $Z_{in} \gg Z_{out}$ condition is not applicable but, instead, radio-frequency circuits are typically engineered to have $Z_{in} = Z_{out} = 50\Omega$. This convention is chosen to ensure maximum power transfer between the circuit stages.)

6 Four-Wire Resistance Measurement

When measuring the resistance R of a highly conductive sample, one must be aware of the following experimental problem: The resistance of the instrumental lead wires used in the measurement may be on the order of (or even greater than) R , thereby introducing an error in the determined value for R . However, by implementing the four-wire resistance measurement technique, which is illustrated in the following diagram, this problem can be eliminated.

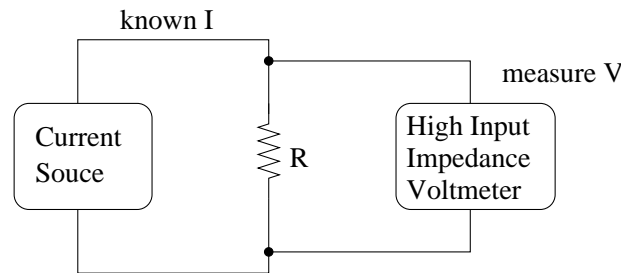


Figure 6: Four-wire resistance measurement.

In this technique, a current source produces a known current I and two wires carry this I through the unknown resistance R . A voltmeter, placed in parallel with R , then measures the voltage V across R . Since the voltmeter has a high input impedance, no significant current passes through its lead wires (and so there is no significant voltage drop across these leads). Thus the measured value for V is the true voltage across the unknown resistor. Then R can be calculated by taking the ratio V/I . Commonly, high-quality digital voltmeters, such as the HP 34401A, can be configured to perform a four-wire resistance measurement.