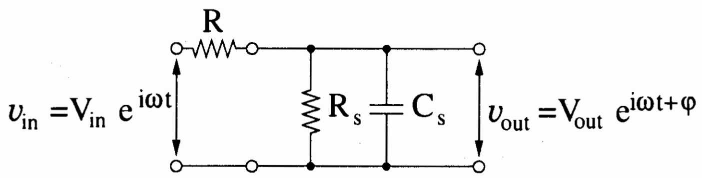
Name: ______________________________

There are **two questions** to complete.

1.



(a) Determine the effective complex impedance $Z_{||}$ of the parallel combination of R_s and C_s.

Now assume you can measure R_s and choose R, in the circuit shown above, such that $R = R_s$. Assume furthermore that you can measure and vary the frequency f of the input signal such that the magnitude of the capacitor's impedance equals the resistance, $|Z_C| = R = R_s$.

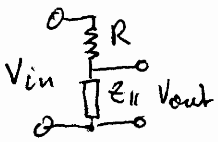
(b) Determine the ratio of the amplitudes V_{out} and V_{in}, *i.e.* the ratio of $|v_{out}|$ and $|v_{in}|$.

(c) What is the value of the capacitance C_s in terms of the measured quantities R and f?

a) $Z_R = R_S$; $Z_C = \dfrac{1}{i\omega C_S}$; $\dfrac{1}{Z_{||}} = \dfrac{1}{Z_R} + \dfrac{1}{Z_C} = \dfrac{1}{R_S} + i\omega C_S = \dfrac{1 + i\omega R_S C_S}{R_S}$

$\hookrightarrow$ $\boxed{Z_{||} = \dfrac{R_S}{1 + i\omega R_S C_S}}$ and $|Z_{||}| = \dfrac{R_S}{\sqrt{1 + (\omega R_S C_S)^2}}$

b) The circuit can be viewed as a voltage divider



$$\frac{|V_{out}|}{|V_{in}|} = \left| \frac{Z_{||}}{R + Z_{||}} \right|$$

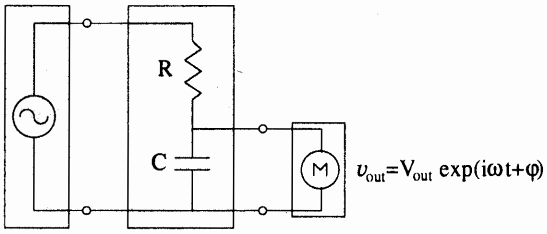
Given $R = R_S$ and $f = f^*$ such that $|Z_C| = \dfrac{1}{2\pi f^* C_S} = R$

we get $Z_{||} = \dfrac{R}{1+i}$, therefore $\dfrac{|V_{out}|}{|V_{in}|} = \left| \dfrac{R/1+i}{R + \frac{R}{1+i}} \right| = \left| \dfrac{1}{2+i} \right| = \dfrac{1}{\sqrt{5}}$

c) $\boxed{C_S = \dfrac{1}{2\pi f^* R}}$

2.

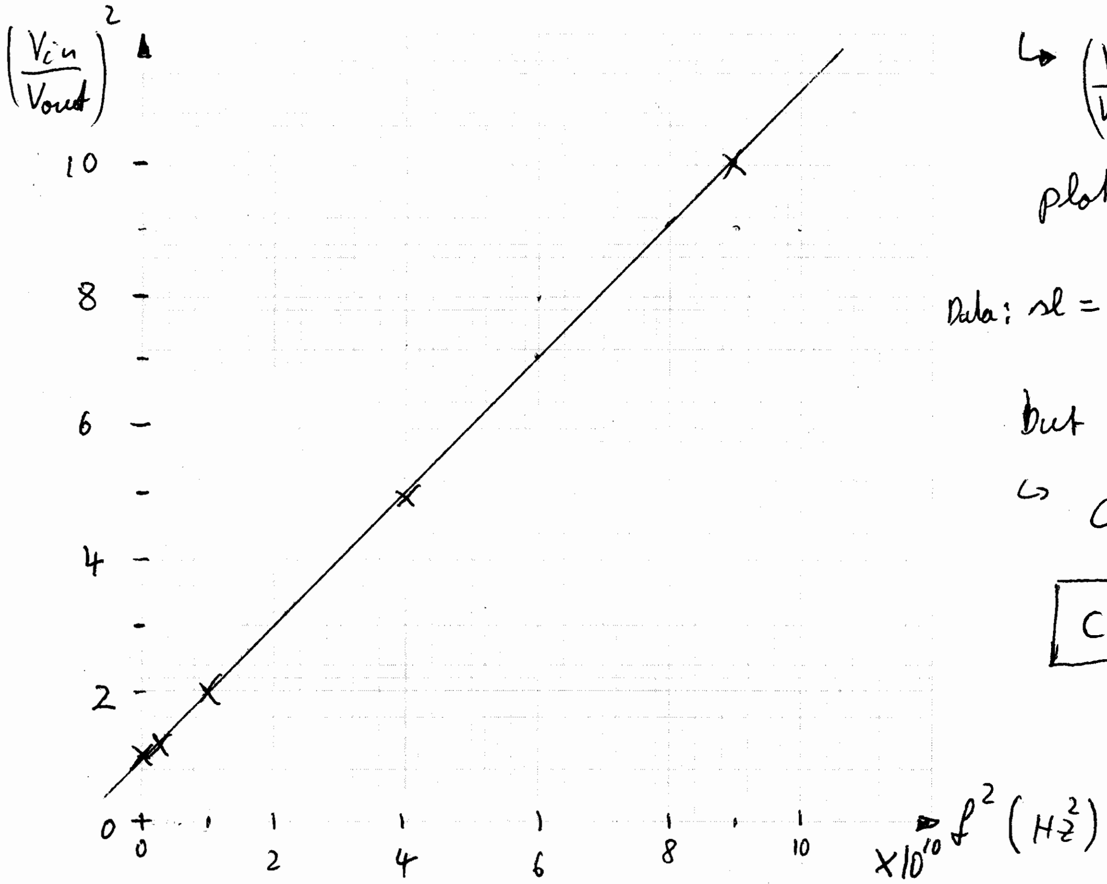
f^2 Hz^2	f (kHz)	$V_{out}(V)$	$\left(\frac{10V}{V_{out}}\right)^2$
0	0	10.0	1
$1\cdot10^8$	10	10.0	1
$0{,}25\cdot10^{10}$	50	8.90	1,26
$1\cdot10^{10}$	100	7.11	1,98
$4\cdot10^{10}$	200	4.49	4,96
$9\cdot10^{10}$	300	3.16	10,0



Given $R = 1$ kΩ and V_{in}=10 V, graph the above data on a straight line plot and determine the value of the capacitance C.

Voltage divider with $Z_1 = R$ $\quad Z_2 = \frac{1}{i\omega C}$

$$\frac{V_{out}}{V_{in}} = \left|\frac{Z_2}{Z_1+Z_2}\right| = \left|\frac{1/i\omega C}{R + 1/i\omega C}\right| = \frac{1}{|i\omega RC+1|} = \frac{1}{\sqrt{(\omega RC)^2+1}}$$



$\hookrightarrow \left(\frac{V_{in}}{V_{out}}\right)^2 = 1 + (2\pi RC)^2 \cdot f^2$

plot $\left(\frac{V_{in}}{V_{out}}\right)^2$ vs. f^2

Data: $sl = \frac{[10-1]}{9\cdot10^{10}\ Hz^2} = 1\cdot10^{-10}\ s^2$

but $sl = (2\pi RC)^2$

$\hookrightarrow C = \frac{\sqrt{sl}}{2\pi R} = \frac{1\cdot10^{-5}\ s}{2\pi\cdot1\cdot10^3\ \Omega}$

$C \approx 1{,}6\ nF$