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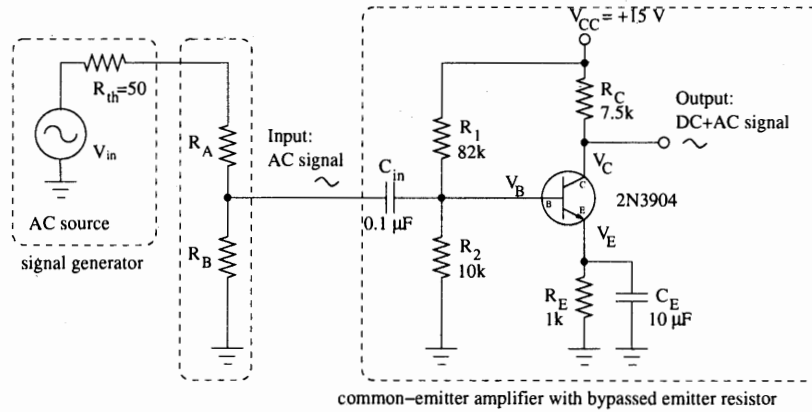


Figure 1: The common-emitter amplifier with bypassed resistor: $R_C = 7.5 \text{ k}\Omega$, $R_E = 1.0 \text{ k}\Omega$, $R_1 = 82 \text{ k}\Omega$, $R_2 = 10 \text{ k}\Omega$, $V_{CC} = +15 \text{ V}$, 2N3904 transistor, $10 \text{ }\mu\text{F}$ emitter capacitor, $0.1 \text{ }\mu\text{F}$ blocking capacitor (on the input). Also shown are the $50 \text{ }\Omega$ signal generator and the voltage divider.

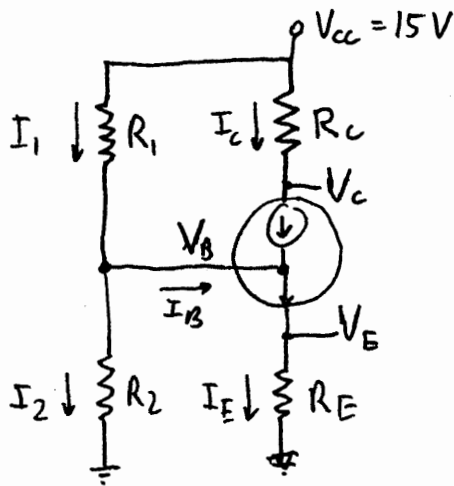
1. For the common-emitter amplifier circuit shown in Fig. 1 consider the DC-voltages and currents when no AC-input is present. Draw the equivalent “DC-circuit” and calculate the quiescent voltages V_B and V_C and the quiescent collector current I_C .
2. Derive the (small-signal) gain of the common-emitter amplifier shown in Fig. 1 (Assume, for simplicity, the circuit is biased to $V_C \simeq 0.5 V_{CC}$). Note that the small signal gain refers to AC-voltages for which above amplifier becomes a grounded-common emitter amplifier. Comment on the role of the quiescent collector current (the DC-current I_C).
3. In order to make the input signal small enough we will use a 100:1 voltage divider in between the signal generator ($50 \text{ }\Omega$ output impedance) and the common emitter amplifier (with the resistance values shown in Fig. 1). Suggest resistor values R_A and R_B of the voltage divider that would work for this purpose and discuss why your choice is reasonable.

[Hint: Show (a) that the resulting AC-voltage is reduced by a factor of 100 and (b) that in this three-stage circuit $Z_{out}^{previous\ stage} \ll Z_{in}^{next\ stage}$ for both cases. Part (b) requires that you calculate the input and the output impedance of the voltage divider as well as the AC-input impedance of the common-emitter amplifier (see example 6, page 16, handout).]

1.) For the DC-analysis we can use the simple current amplifier model with the assumption $\beta \gg 1$ (i.e. $\beta \rightarrow \infty$).

Then $I_B \approx 0 \rightarrow I_E \approx I_C$ and $I_1 \approx I_2$; also $V_{BE} = 0.6V = \text{const.}$

Equip. Circuit :



The voltage divider on the left determines V_B
 $I_1 \approx I_2$
 $(R_1 + R_2)^{-1} V_{CC} \approx R_2^{-1} V_B$

$$\hookrightarrow V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{10k\Omega}{82k\Omega + 10k\Omega} \cdot 15V$$

$$\boxed{V_B = 1.6V}$$

The voltage drop $V_{BE} = 0.6V$ in the transistor yields V_E

$$\boxed{V_E = V_B - 0.6V = 1.0V}$$

Therefore, the emitter current is

$$I_E = V_E / R_E = \frac{1.0V}{1.0k\Omega}$$

$$\boxed{I_E = 1mA}$$

With the $\beta \rightarrow \infty$ approx.: $I_C \approx I_E$

$$\boxed{I_C = 1mA}$$

and finally

$$V_{CC} - V_C = I_C R_C$$

$$\hookrightarrow V_C = V_{CC} - I_C R_C = 15V - V_E \frac{R_C}{R_E} = 15V - 1.0V \cdot \frac{7.5}{1.0}$$

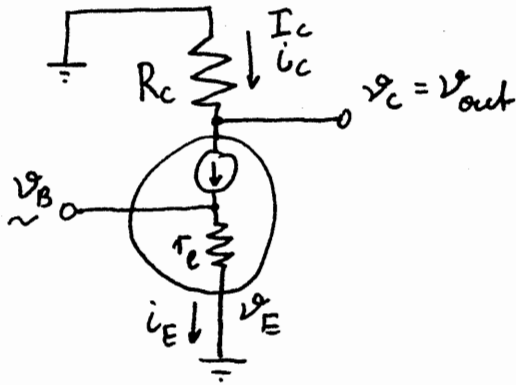
$$\boxed{V_C = 7.5V}$$

or $V_C = 7.3V$ if you use $V_C = V_{CC} - \left[\frac{R_2}{R_1 + R_2} V_{CC} - 0.6V \right] \cdot \frac{R_C}{R_E}$

2.) Use the transconductance amplifier model with $I_B = 0$; $I_C = I_E$ (this is the total current, meaning $i_c = i_E$ holds for the AC-currents)

Assume the input stage is designed correctly. Then $V_B = V_{in}$.

For AC-voltages, the equivalent circuit is. Note that the emitter is grounded:



emitter is grounded:

$$V_E = 0$$

$$\hookrightarrow V_{BE} = V_B - V_E = V_B = V_{in}$$

$$\text{but } V_{BE} = i_E \cdot r_e$$

$$\Rightarrow i_E = \frac{V_{in}}{r_e}$$

$$\text{Furthermore } i_E = i_c \text{ and } i_c = \frac{0V - v_c}{R_c} = -\frac{v_c}{R_c}$$

$$\hookrightarrow v_{out} = v_c = -i_c R_c = -i_E R_c = -\left(\frac{V_{in}}{r_e}\right) R_c$$

$$\hookrightarrow \text{gain} = \left| \frac{v_{out}}{v_{in}} \right| = \frac{R_c}{r_e} \quad \text{with } r_e = \frac{25\text{mV}}{I_c^{\text{total}}}$$

$$\boxed{\text{gain} = \frac{R_c I_c^{\text{total}}}{25\text{mV}}}$$

For small signals v_{in} : $I_c^{\text{total}} = I_c^{\text{DC}} + i_c \approx I_c^{\text{DC}}$ (the quiescent current)

$$\text{Now } I_c^{\text{DC}} = \frac{V_{cc} - V_c}{R_c}. \text{ With } V_c = \frac{V_{cc}}{2}, \text{ we get } I_c^{\text{DC}} \cdot R_c = \frac{V_{cc}}{2} = 7.5\text{V}$$

$$\text{Therefore } \text{gain} \approx \frac{R_c I_c^{\text{DC}}}{25\text{mV}} = \frac{7.5\text{V}}{25 \cdot 10^{-3}\text{V}} = \boxed{300}$$

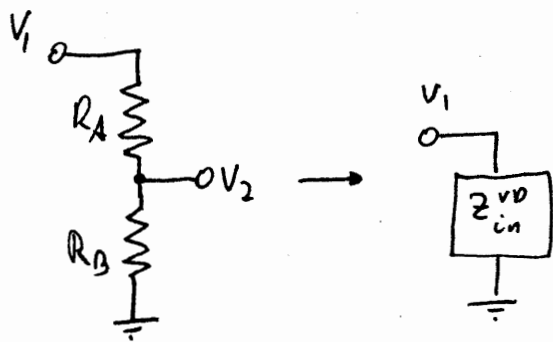
The gain depends on the value of the current I_c . For small input signals this means that one can change the gain by adjusting the quiescent current I_c^{DC} .

3.) No DC-currents will be passed to the amplifier due to the blocking capacitor \rightarrow need to consider AC only.
 We need to figure out the input and output impedances of the three stages.

I) Output impedance of the signal generator is given:

$$Z_{out}^{SG} = 50\Omega$$

II) Input impedance of the voltage divider:

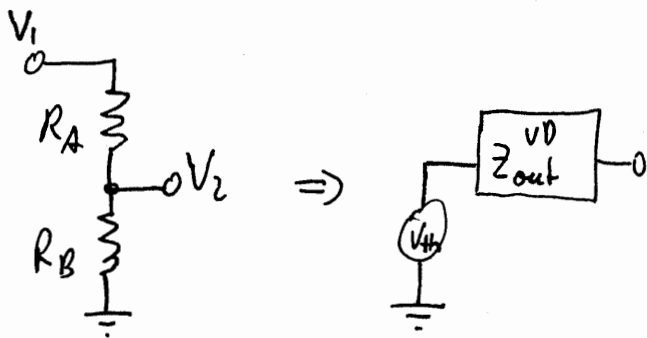


When nothing (no load) is attached to the output V_2 , then

$$I = V_1 / (R_A + R_B)$$

$$\hookrightarrow Z_{in}^{VD} = R_A + R_B$$

III) Output impedance of the voltage divider



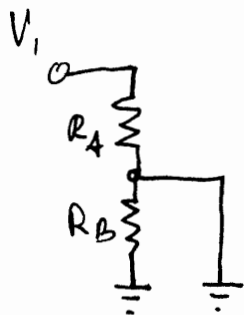
First, we determine $V_{th} = V_2$, the output voltage when no load is attached.

$$V_2 = \frac{R_B}{R_A + R_B} V_1 \quad (*)$$

Second, we determine $I_{short} = \frac{V_1}{R_A}$

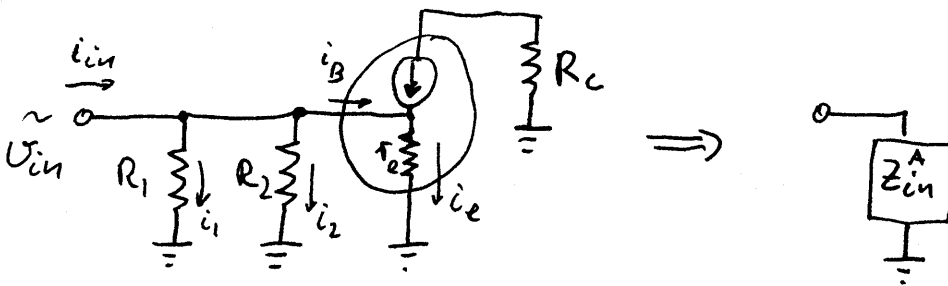
$$\text{Then } Z_{out}^{VD} = \frac{V_{th}}{I_{short}} = \frac{R_B V_1}{R_A + R_B} \cdot \frac{R_A}{V_1} = \frac{R_A R_B}{R_A + R_B},$$

$$\text{i.e. } Z_{out}^{VD} = R_A \parallel R_B$$



3.) IV) AC-input impedance of the transistor based common emitter amplifier:

The AC-equivalent circuit is



We are using Ohm's law $Z_{in}^A = \frac{v_{in}}{i_{in}}$.

$$i_{in} = i_1 + i_2 + i_B \quad i_1 = v_{in}/R_1 \quad i_2 = v_{in}/R_2$$

Furthermore, the emitter-pin is grounded $v_E = 0$. Using the transconductance model we get

$$r_e \cdot i_E = v_{BE} = v_B - v_E = v_B = v_{in}$$

and because $i_E = (\beta + 1) i_B$, we have $i_B = \frac{v_{in}}{(\beta + 1) r_e}$.

Let us estimate $(\beta + 1) r_e$:

$$r_e = \frac{25 \text{ mV}}{I_c} \approx \frac{25 \text{ mV}}{I_c^{DC}} = 25 \Omega$$

↑
1mA from problem 1

with $\beta \approx 100 \rightarrow (\beta + 1) r_e \approx 2.5 \text{ k}\Omega$

Note that you need to first determine i_E and use i_E to get i_B !
[$i_B \neq v_{in}/r_e$!]

Now putting it all together:

$$Z_{in}^A = v_{in} \cdot \left(\frac{v_{in}}{R_1} + \frac{v_{in}}{R_2} + \frac{v_{in}}{(\beta + 1) r_e} \right)^{-1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{(\beta + 1) r_e} \right)^{-1}$$

$$= \left(\frac{1}{10 \text{ k}\Omega} + \frac{1}{82 \text{ k}\Omega} + \frac{1}{2.5 \text{ k}\Omega} \right)^{-1} \approx 20 \text{ k}\Omega$$

$Z_{in}^A = 2.0 \text{ k}\Omega$

(3.) So a 1:100 voltage divider implies [use (*)]

$$\cdot \quad \frac{1}{100} = \frac{V_2}{V_1} = \frac{R_B}{R_A + R_B} \quad \rightarrow \quad R_A = 99 R_B$$

$$\cdot \quad Z_{out}^{SG} \ll Z_{in}^{VD} \quad \text{implies} \quad 50 \Omega \ll R_A + R_B = 100 R_B$$
$$\quad \rightarrow \quad R_B \gg 0.5 \Omega$$

$$\cdot \quad Z_{out}^{VD} \ll Z_{in}^A \quad \text{implies} \quad \frac{R_A R_B}{R_A + R_B} = \frac{99}{100} R_B \ll 2 \text{ k}\Omega$$

\rightarrow We might pick 5% resistors with the following values [you should check that these exist].

$$R_B = 100 \Omega \quad R_A = 100 \text{ k}\Omega$$

$$\text{or } R_B = 51 \Omega \quad R_A = 5.1 \text{ k}\Omega$$