Name: ______________________________

1. Determine an expression for the gain $g = \frac{V_{out}}{V_{in}}$ of the non-inverting amplifier shown in Fig. 1. The triangles indicate the ground (0 V) level, relative to which V_{in} and V_{out} are measured.

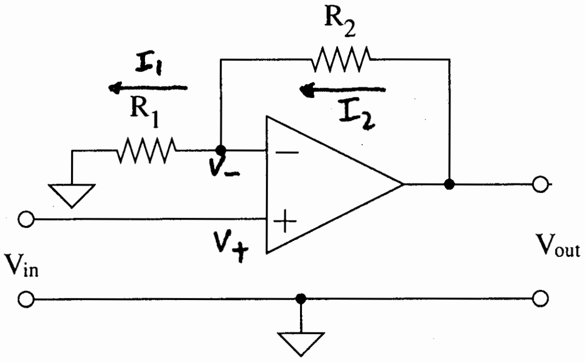

Figure 1: Non-inverting amplifier

Using the Golden Rules

GR I: $V_- = V_+ = V_{in}$

GR II: $I_2 = I_1$

use Ohm's law for the currents I_2, I_1

$$\frac{V_{out} - V_-}{R_2} = \frac{V_-}{R_1}$$

$$\frac{V_{out} - V_{in}}{R_2} = \frac{V_{in}}{R_1}$$

$$\Rightarrow V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

$$\boxed{g = 1 + \frac{R_2}{R_1}}$$

2.

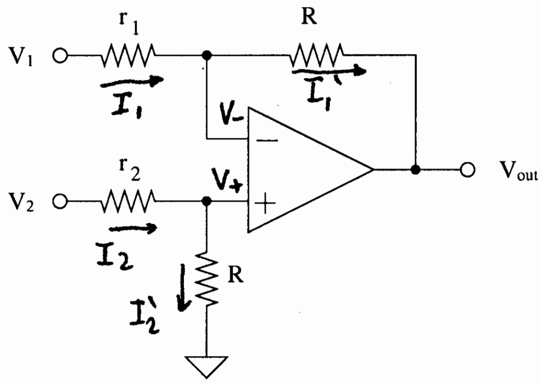

Figure 2: "Making a difference"

For $r_1 = r_2 = r$, the circuit on the left has an output that is the amplified voltage difference of its inputs.

$$V_{out} = \frac{R}{r}(V_2 - V_1)$$

However, measuring differences can be difficult as you will show in this and the next problem.

Show that for $r_1 \neq r_2$

$$V_{out} = \left(\frac{r_1 + R}{r_2 + R}\,\frac{R}{r_1}\right) V_2 - \frac{R}{r_1} V_1.$$

Use the Golden Rules for op-amps:

GR I : $V_- = V_+$

GR II : $I_1 = I_1'$

$I_2 = I_2'$

use Ohm's law $I_1 = I_1' \rightarrow \dfrac{V_1 - V_-}{r_1} = \dfrac{V_- - V_{out}}{R} \rightarrow V_{out} = V_- - \dfrac{R}{r_1}(V_1 - V_-)$

$V_+ = V_-$; $I_2 = I_2' \rightarrow \dfrac{V_2 - V_-}{r_2} = \dfrac{V_-}{R} \rightarrow V_2 = V_-\left(1 + \dfrac{r_2}{R}\right)$

$\hookrightarrow V_- = V_+ = V_2 \dfrac{R}{r_2 + R}$

$$\Rightarrow V_{out} = V_2 \frac{R}{r_2+R}\left(1 + \frac{R}{r_1}\right) - \frac{R}{r_1} V_1$$

$$\boxed{V_{out} = \left(\frac{r_1 + R}{r_2 + R}\,\frac{R}{r_1}\right) V_2 - \frac{R}{r_1} V_1}$$

3. Continuing with the circuit shown in Fig. 2, consider a situation where you were able to pick resistors such that $R = r_1 = r$ holds exactly, so that the resistor values ($= r$) have a standard deviation $\sigma = 0$. In contrast, for r_2 you just took a resistor from the bin, so there is an uncertainty σ_r, *i.e.* $r_2 = r \pm \sigma_r$.

a) Derive an expression for the resulting error term σ_Δ: $V_{out} = \Delta = (V_2 - V_1) \pm \sigma_\Delta$.

b) In lab we use 5% resistors ($\sigma_r/r = 5\%$). If $V_2 = 10$ V, what is the minimum voltage difference $|\Delta| = |V_2 - V_1|$ for which the resulting relative measurement uncertainty is better than 10% ($\sigma_\Delta/|\Delta| < .1$)?

a) From 2: $V_{out} = \frac{r_1 + R}{r_2 + R}\frac{R}{r_1} V_2 - \frac{R}{r_1} V_1 \underset{r=R=r_1}{=} \frac{2r}{r_2+r} V_2 - V_1 \underset{r_2=r}{=} V_2 - V_1 = \Delta$

To keep track of error define $x = r \pm 0$ and $y = r_2 = r \pm \sigma_r$. We want to know the absolute uncertainty σ_Δ of

$\Delta(x,y) = \frac{2x}{x+y} V_2 - V_1$ where $x = y = r$

From handout: $\sigma_\Delta = \sqrt{\left(\frac{\partial\Delta}{\partial x}\right)^2 \cdot (0)^2 + \left(\frac{\partial\Delta}{\partial y}\right)^2 \sigma_r^2} = \left|\frac{\partial\Delta}{\partial y}\right| \sigma_r$

$\left.\frac{\partial\Delta}{\partial y}\right|_{x=y=r} = \left.\frac{-2x}{(x+y)^2} V_2\right|_{x=y=r} = -V_2/2r \rightarrow \boxed{\sigma_\Delta = \frac{\sigma_r V_2}{2r}}$

b) $\frac{\sigma_r}{r} = 5\%$; Want $\frac{\sigma_\Delta}{|\Delta|} < 10\%$

$\hookrightarrow \frac{\sigma_r}{r} \cdot \frac{V_2}{2} < 10\% \cdot |\Delta| \rightarrow \frac{5\%}{10\%} \frac{10\,V}{2} < |\Delta| \rightarrow \boxed{|\Delta| > 2.5\,V}$

If the voltage difference at the two input ports falls below 25% of V_2, the measurement uncertainty becomes unacceptable. Unless the resistors are matched precisely, this is a <u>terrible</u> circuit for measuring $V_2 - V_1$. There exist special circuits called "Instrument Amplifiers" that are designed for the purpose of measuring $V_2 - V_1$ with good precision.