Name: ______________________________

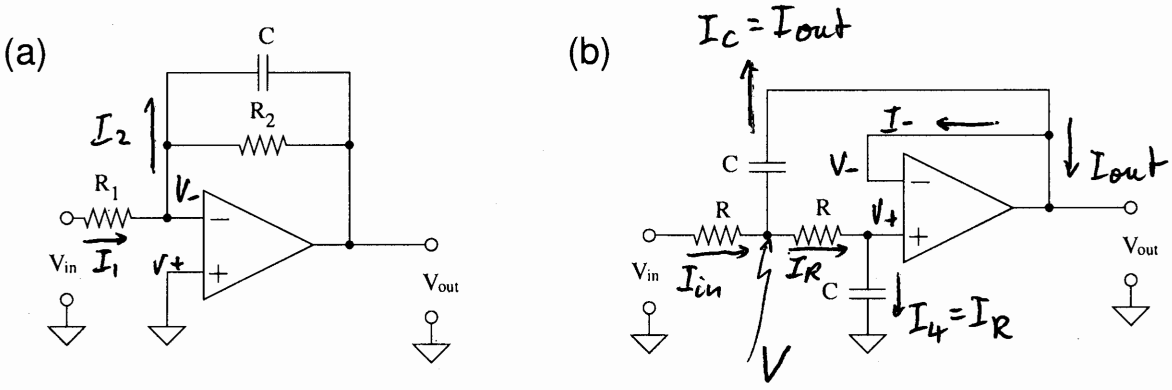
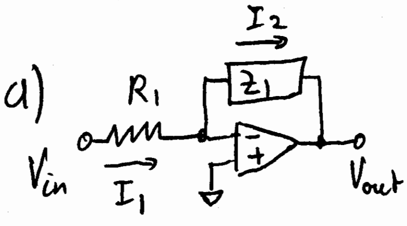


Figure 1: (a) First order active low-pass filter. (b) Sallen-Key second order low-pass filter.

1. Derive Eq. (8) and Eq. (9) in the handout, the equations for the first order low-pass active filters described in the handout for this lab and shown in Fig. 1.

a)



GRI: $V_+ = V_- = 0$

GRII: $I_1 = I_2 \Rightarrow V_{in}/R_1 = -\frac{V_{out}}{Z_1}$

$\hookrightarrow H = \frac{V_{out}}{V_{in}} = -Z_1/R_1$ (inverting ampl.)

Now $Z_1 = \frac{R_2 \cdot \frac{1}{i\omega C}}{R_2 + \frac{1}{i\omega C}} = \frac{R_2}{1+i\omega R_2 C}$

$$\boxed{|H| = \frac{R_2}{R_1}\frac{1}{\sqrt{1+(\omega R_2 C)^2}}}$$

b) GRI: $V_+ = V_- = V_{out}$

GRII: $I_R = I_4$ & $I_- = 0 \rightarrow I_C = I_{out}$

- From $I_R = I_4$ we get $\frac{V}{R+\frac{1}{i\omega C}} = \frac{V_{out}}{1/i\omega C}$

So $V = (1+i\omega RC)V_{out}$ or $(V - V_{out}) = (i\omega RC)V_{out}$

- From $I_{in} = I_R + I_{out}$ we get

$$\frac{V_{in}-V}{R} = \frac{V_{out}}{1/i\omega C} + \frac{V-V_{out}}{1/i\omega C}$$

$$V_{in} = (i\omega RC)[V_{out} + (V - V_{out})] + V$$

$$\overset{!}{=} (i\omega RC)(1+i\omega RC)V_{out} + (1+i\omega RC)V_{out}$$

$$= (1+i\omega RC)^2 V_{out}$$

Therefore $|H(\omega)| = |V_{out}/V_{in}| = \frac{1}{|(1+i\omega RC)^2|}$

$$|H| = \frac{1}{|1+i\omega RC|\cdot|1+i\omega RC|} = \frac{1}{\left(\sqrt{1+(\omega RC)^2}\right)^2}$$

$$\hookrightarrow \boxed{|H(\omega)| = \frac{1}{1+(\omega RC)^2}}$$

2. Derive an expression for the 3 dB-point (f_{3dB}) for each of the filters described by Eqns. (8) and (9) in terms of their resistances and capacitances.

a) first-order LP filter:

$$H_{max} = |H(\omega=0)| = \frac{R_2}{R_1}$$

$$H_{max}/\sqrt{2} = \frac{R_2}{\sqrt{2}R_1} := \frac{R_2}{R_1}\frac{1}{\sqrt{1+(\omega R_2 C)^2}}$$

$$\hookrightarrow 2 = 1 + (\omega R_2 C)^2 \Rightarrow \omega R_2 C = 1$$

$$\hookrightarrow \boxed{f_{3dB} = \frac{1}{2\pi R_2 C}}$$

b) Sallen-Key filter:

$$H_{max} = |H(\omega=0)| = 1$$

$$H_{max}/\sqrt{2} = \frac{1}{1+(\omega RC)^2} \Rightarrow \sqrt{2} = 1 + (\omega RC)^2$$

$$\hookrightarrow \omega RC = \sqrt{\sqrt{2}-1}$$

$$\hookrightarrow \boxed{f_{3dB} = \frac{\sqrt{\sqrt{2}-1}}{2\pi RC} \approx \frac{0.64}{2\pi RC}}$$

3. Assume that you have a lot of 0.01μF capacitors. Commercial resistors have values $m \times 10^n\ \Omega$, where n may range from 0 to 6 and available values for m are 10, 11, 12, 13, 15, 16, 18, 20, 22, 24, 27, 30, 33, 36, 39, 43, 47, 51, 56, 62, 68, 75, 82 and 91. To make a unity-gain first-order low-pass RC active filter with $f_{3dB} = 1600$ Hz, what value resistors should you choose for R_1 and R_2 (Fig. 1a)? To construct a second-order Sallen-and-Key low-pass filter with the same 3 dB-frequency (Fig. 1b), what value R should you choose?

a) $f_{3dB} = \dfrac{1}{2\pi R_2 C}$; Try $C = 0.01\mu F$

$\hookrightarrow R_2 = \dfrac{1}{2\pi \cdot 1600\ s^{-1} \cdot 0.01\mu F} = 9947\,\Omega \approx 10\,k\Omega$

b) $f_{3dB} \approx \dfrac{0.64}{2\pi RC}$; Try $C = 0.01\mu F$

$\hookrightarrow R \approx 0.64 \cdot R_2 = 6.4\,k\Omega$

Could try first a 6.2kΩ resistor and add a 200Ω resistor in series, if necessary.