Name: _____

1.   By writing out the truth tables involved, prove deMorgan's theorem in its two forms:

a) $\overline{A + B} = \overline{A} \cdot \overline{B}$ [NOT (A OR B) = (NOT A) AND (NOT B)]

b) $\overline{A \cdot B} = \overline{A} + \overline{B}$ [NOT (A AND B) = (NOT A) OR (NOT B)]

2.   We want to prove the assertion that NAND is universal, *i.e. all binary functions, of any number of arguments, can be expressed in terms of NAND functions alone.* We will skip some preliminary steps of the proof, and just take as given that

1. All binary functions of three or more arguments can be expressed as combinations of binary functions of two arguments only. (One can show this by a tedious but elementary method of boolean function reduction.)

2. As shown in Simpson on page 538, there are exactly 16 binary functions with two input arguments. By thinking about these sixteen functions, as discussed in table 12.7. in Simpson on page 539, you may also convince yourself that every of the 16 functions is expressible in terms of AND, OR and NOT functions.

Therefore we now need to show only that AND, OR and NOT can all be expressed in terms of NAND alone.

Example: To show that a NAND gate can be used to implement the NOT function, observe that

$$\text{NAND(A, A)} = \text{NOT(A AND A)} = \text{NOT (A)}.$$

So we can implement the NOT gate as
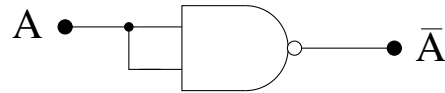
a) Show how to implement the AND function using the proper configuration of several NAND gates.

b) Use deMorgan's theorem to show how to make an OR gate by the proper configuration of several NAND gates.
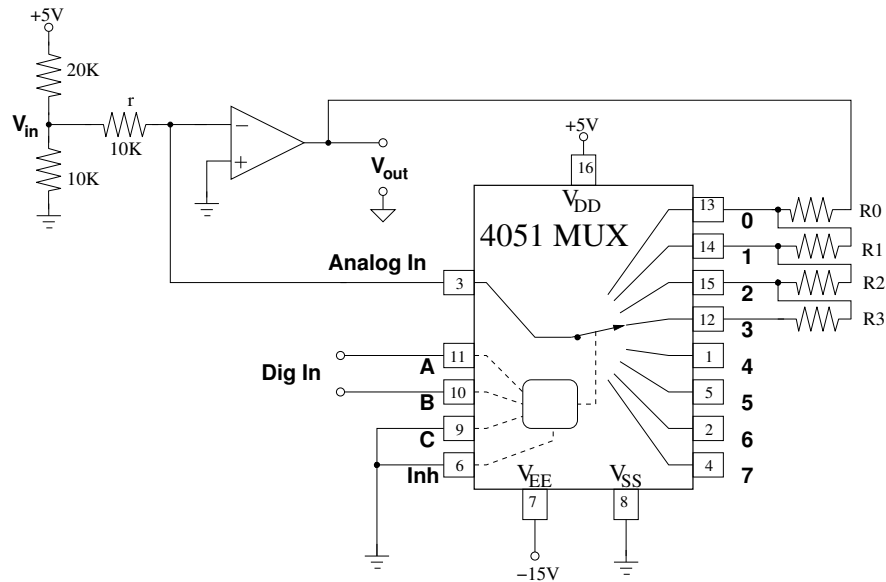
3.   An inverting amplifier with digital gain control.

Figure 1: The 4051 analog multiplexer: "rotary control" type schematic of 4051 used to implement an inverting amplifier with digital gain control.

a) What is the value of V_{in} in the circuit shown?

b) How should you pick R0, R1, R2, R3 to achieve digitally controllable gains of 1, 2, 3, and 4? (gain $= V_{out}/V_{in}$)

c) What are the digital input states for A and B that result in gains of 1, 2, 3, and 4?