APPARATUS AND DEMONSTRATION NOTES

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Experimental demonstration of Doppler spectral broadening using the PC sound card

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The Doppler broadening of audio spectral lines is demonstrated using a simple experimental setup. The sound from a vibrating loudspeaker is received by a microphone that is connected to a computer sound card. A data-acquisition program is used to acquire the sound data. Fourier analysis is done to obtain the spectral power density of the sound received by the microphone. The results demonstrate spectral broadening similar to that which takes place in atomic and molecular spectroscopy. The experiment demonstrates both the Doppler effect and spectral broadening in the undergraduate physics lab.

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I. INTRODUCTION

The Doppler effect associated with sound propagation is well known. Consider a source of sound S with frequency f that moves with velocity v_s relative to a laboratory. The frequency f' heard by an observer O who moves with velocity v_0 relative to the laboratory is given by

$$f' = \frac{c - \mathrm{v}_O}{c - \mathrm{v}_S} f, \tag{1}$$

where c is the speed of sound. The sign convention is to consider v_0 and v_s as positive if their directions are parallel to the direction of wave propagation and negative if they move opposite to the direction of wave propagation. If the observer is stationary, Eq. (1) reduces to

$$f' = \frac{c}{c - \mathrm{v}_S} f. \tag{2}$$

An experiment for demonstrating this effect using a PC-sound card as a data acquisition tool has been described by Bensky and Frey.[1] The experiment involves the recording of sound from a moving loudspeaker that is mounted on an air-track cart. A stationary microphone receives the sound from the moving loudspeaker. The sound is recorded and later analyzed by a computer to determine the frequency changes due to the Doppler effect.

An interesting consequence of the Doppler effect is the Doppler broadening of spectral lines. If a gaseous source is at high temperatures, a spread in the frequency of a given transition is produced by the atoms' randomly oriented velocities with respect to a spectrograph.[2]

For simplicity, we assume that a stationary source and a spectrograph are separated by a distance D. Let us further assume that the source is oscillating with a frequency f_0 and constant amplitude A_0. The instantaneous value of the wave amplitude $A(t)$ received by the spectrograph can be written as

$$A(t) = A_0 \sin 2\pi f_0 t. \tag{3}$$

If the source vibrates mechanically along the line between S and O with frequency ν and amplitude X about its original stationary position $x=0$, the source position $x(t)$ at any time is given by

$$x(t) = X \sin 2\pi\nu t, \tag{4}$$

and its velocity v_S is

$$\mathrm{v}_S = \frac{dx}{dt} = 2\pi X\nu \cos 2\pi\nu t. \tag{5}$$

For this situation, Eq. (3) becomes, using Eq. (2),

$$A(t) = A_0 \sin\left(2\pi f_0 \frac{c}{c - 2\pi X\nu \cos 2\pi\nu t}\right). \tag{6}$$

This result indicates that a single frequency f_0 emitted by the source S when detected by the observer or the spectrograph will not be a sharp line at frequency f_0. Instead, the observed spectrum will extend over the range between the upper and lower limits

$$f_0\left(\frac{c}{c-(2\pi X\nu/\sqrt{2})}\right) > f_0^{\copyright} > f_0\left(\frac{c}{c+(2\pi X\nu/\sqrt{2})}\right). \quad (7)$$

In Eq. (7) the root-mean-square value of the source velocity is substituted for the actual instantaneous velocity in Eq. (6). It is clear that the broadening effect is related to both the source oscillation amplitude and the source oscillation frequency.

One further effect that needs to be considered is that in practice, no real measurement instrument can reproduce a sharp, delta-function-like spectrum. A finite width is always associated with a measured peak in a frequency spectrum. Real spectral analysis instruments produce peaks with finite values of the full-width-at-half-maximum (FWHM). For a source at a single well-defined frequency, the value of the FWHM of the detected peak in the frequency spectrum is a measure of the instrument resolution.

It is common to describe the spectral power-density of the peaks obtained by a spectrograph in a wide range of physics applications by Gaussian shapes of the form

$$P(f) = Ke^{-(f-f_0)^2/\sigma^2}, \quad (8)$$

where $P(f)$ is the power density at a particular frequency f, K is a normalization constant, f_0 is the frequency at the peak, and σ defines the width of the peak. If we use Eq. (8), we find the full-width-at-half-maximum Δf to be

$$\Delta f = \tfrac{1}{2}\sigma(\ln 2). \quad (9)$$

Equation (9) defines the width of a spectral peak for a stationary source. The parameter σ is determined by the resolution of the instrument.

An approximate expression for the FWHM of a measured peak due to the combined effects of instrumental resolution and Doppler broadening can be obtained by adding another term to Eq. (9). This term is equal to the difference between two mean frequencies of the vibrating source as detected by the spectrograph. The first is the mean frequency detected during the first half of the vibration cycle when the source is approaching the spectrograph, $f_0/(1-\pi Xv/c\sqrt{2})$. The second is the mean frequency detected during the second half of the vibration cycle when the source is moving away from the spectrograph, $f_0/(1+\pi av/c\sqrt{2})$. To a first approximation, the new FWHM can be written as

$$\Delta f_D = \tfrac{1}{2}\sigma(\ln 2) + [f_0/(1-\pi X\nu/c\sqrt{2}) - f_0/(1+\pi X\nu/c\sqrt{2})], \quad (10)$$

which can be simplified to yield

$$\Delta f_D = \frac{1}{2}\sigma(\ln 2) + \frac{2f_0(\pi X\nu/c\sqrt{2})}{1-(\pi X\nu/c\sqrt{2})^2}. \quad (11)$$

II. EXPERIMENTAL SETUP AND SOFTWARE

The following experiment demonstrates this broadening effect using an ordinary general-purpose computer sound-card and a small loudspeaker attached to the end of a vibrating metal rod. The experimental setup is shown in Fig. 1. A 0.5 W loudspeaker is connected to a Metrix BF 817 wave generator. The frequency of the wave generator is set to about 3250 Hz. This frequency produces optimum matching between the oscillator and the loudspeaker. The loudspeaker is rigidly fixed to one end of a vertical metal ruler-bar. The other end of the bar is rigidly fixed to the laboratory bench. This setup allows the speaker to vibrate along the direction of the microphone, which is positioned about 30 cm away from the static position of the speaker. The microphone is connected directly to the computer sound card in the usual manner.

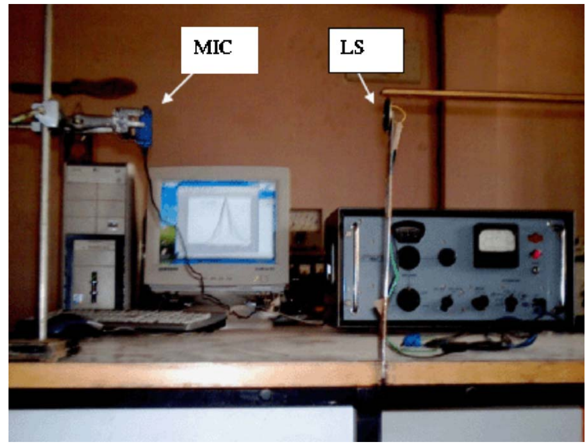

Fig. 1. General view of the experimental setup showing the loudspeaker (LS) and the microphone (MIC).

The Matlab data acquisition toolbox is used to acquire data from the sound card. The sound card operates as an analog-to-digital converter. A Matlab program was written to acquire the sound data and perform the fast Fourier transformation. The program listing is given in Appendix A.

III. EXPERIMENTAL PROCEDURE

The purpose of the experiment is to demonstrate the effect of the speaker's speed on the frequency spectrum of the sound received by the microphone (Doppler broadening). The first step is to check the instruments' reliability by turning on the speaker, running the program, and analyzing the signal received by the microphone over the entire frequency range. The frequency spectrum is shown in Fig. 2. A single peak appears at 3250 Hz. Because our wave generator can produce a saw tooth waveform in addition to a sinusoidal

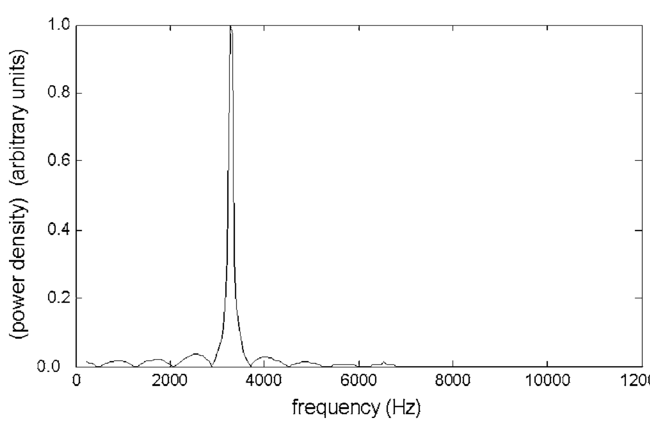

Fig. 2. Plot of the signal power density vs frequency for a 3250 Hz sinusoidal signal applied to the line-in port of the sound card.

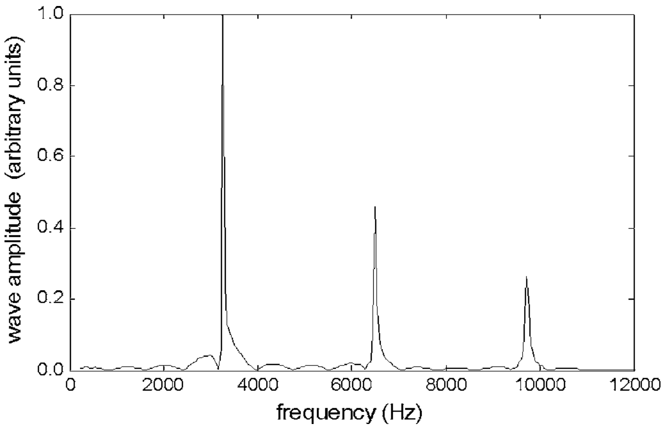

Fig. 3. Plot of the signal amplitude vs frequency for a 3250 Hz saw tooth signal applied to the line-in port of the sound card. This signal contains a second harmonic component and a third harmonic component at 6500 and 9750 Hz, respectively.

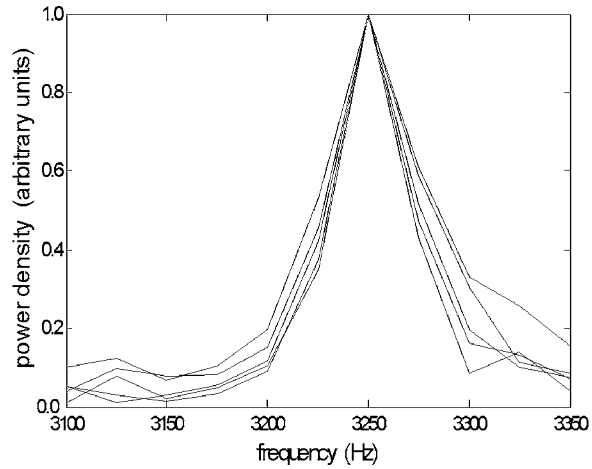

Fig. 4. Successive peaks of a 3250 Hz sinusoidal signal. The outermost peak represents zero time and the other peaks are separated by 0.063 s. The innermost peak represents a time of 0.252 s. The reduction in peak width is due to the vibrational damping of the loudspeaker.

one, a further check is performed. This check involves applying the saw tooth waveform to the speaker. The spectrum of this signal is shown in Fig. 3 where we have plotted the wave amplitude instead of the wave power density. The amplitude is proportional to the square root of the power density. The relative heights of the 3250, 6500, and 9750 Hz harmonic peaks are 1, 0.5, and 0.33, respectively, and are equal to the Fourier harmonic coefficients of the saw tooth wave form.[3]

Once the reliability of the system is established using a stationary speaker, we can do the vibrating speaker experiment. We first select the proper frequency window of interest to study the peak in more detail. For this purpose, the software is set to plot the data in the frequency range 3100−3350 Hz instead of the entire audio frequency range. The software is modified to run for five successive times by putting the whole program inside a loop (see Appendix B).

The rod holding the speaker is displaced a certain distance and allowed to vibrate and the program is run. Plots of the successive peaks will start to appear on the screen. The first peak will be the one with the greatest width. Successive peaks will show smaller widths because they represent lower root-mean-square speeds of the loudspeaker due to damping of the speaker's motion. The average time interval between the time of successive peaks is approximately equal to the time the computer needs to acquire the sound data. This time is equal to the number of data points acquired during each run divided by the sampling rate and is typically of the order of 0.1 s (neglecting CPU time). The results of two experiments done with 500 and 1000 data points are shown in Figs. 4 and 5. The effect of source vibration is clear in both figures. By comparing the two cases we see that the reduction in the peak broadening rate is smaller when the sampling rate is 500 samples per second. The time interval between any two successive peaks is approximately 0.063 s. For the case shown in Fig. 5, this time is 0.125 s. This latter case corresponds to a greater degree of damping and thus has greater differences in the root-mean-square of the source velocity, resulting in greater differences between successive peak widths.

To study the damping of the vibrations, the logarithm of the FWHM of the power density is plotted against time, as shown in Fig. 6. The plot can be fitted to a straight line with slope $\alpha = -1.8/\text{s}$. This plot shows that the vibration damping is exponential in time as expected for ordinary frictional damping.[4] The slope corresponds to a reduction of the vibration amplitude of 99% (to almost a standstill) over a duration of 2.6 s. Although we have no experimental means to confirm this value accurately, visual observations suggest that such a reduction is taking place over a period of a few seconds. The intercept of the fitted line has no physical importance because it is a measure of the initial displacement of the rod *X*.

A further illustration of Doppler broadening can be seen by acquiring a data set over a time that is so short that damping may be neglected. The speaker is displaced by a measured distance from its equilibrium position and then released at the same moment when the program starts to acquire data. The root-mean-square velocity in this case is proportional to the initial displacement. This procedure will produce a greater amount of broadening for large displacements. The results for initial displacements of 1, 2, 3, 4, and

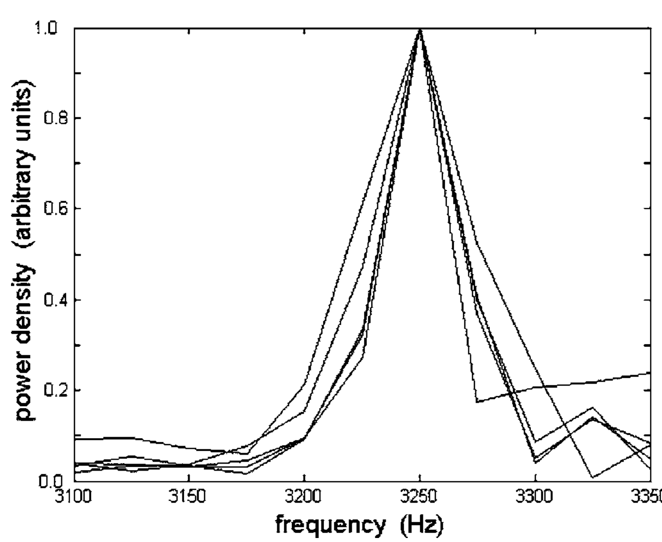

Fig. 5. Results of the same 3250 Hz peak analysis of Fig. 4 using time intervals of 0.125 s. The innermost peak represents a time of 0.5 s.

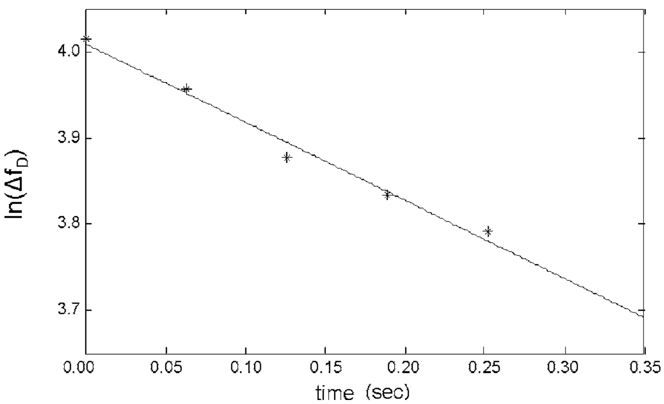

Fig. 6. Plot of the natural logarithm of the full-width-at-half-maximum for the peaks in Fig. 4 as a function of time. The solid straight line represents a linear fit to the data. The fitted line has a slope of $-1.8/\mathrm{s}$, which represents the damping factor.

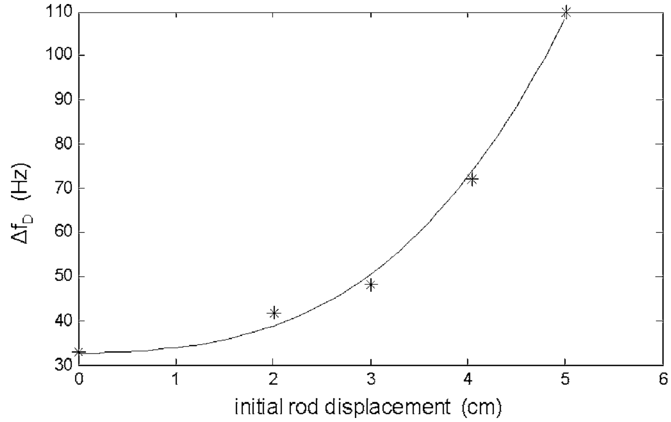

Fig. 8. Plot of the full-width-at-half-maximum for the peaks in Fig. 7 vs the corresponding initial displacements. The solid line represents a fit of the data of Fig. 7 to Eq. (12).

5 cm are plotted in Fig. 7. This type of experimentation can be tricky when performed by one student. Two students can obtain better results if they synchronize the two events.

The FWHM of the peaks for the five different initial displacements are plotted against the rod displacement in Fig. 8. The initial rod displacement is proportional to the vibration amplitude X. If we keep the rod vibration frequency ν constant, Eq. (11) can be approximated for $\pi\nu/c\sqrt{2} \ll 1$ as

$$\Delta f_D = \frac{1}{2}\sigma \ln 2 + \frac{2f_0 BX}{1-(BX)^2} \approx \frac{1}{2}\sigma \ln 2 + (BX + B^3X^3), \tag{12}$$

$$B \equiv \frac{4f_0\pi\nu}{c\sqrt{2}}. \tag{13}$$

The data in Fig. 8 are fitted to Eq. (12) and good agreement between the fitted curve and experimental results is observed. The fit value of $\frac{1}{2}\sigma \ln 2 = 32.5$ Hz corresponds to the peak width for a stationary source, which is a measure of the instrument resolution (the sound card in this case). We

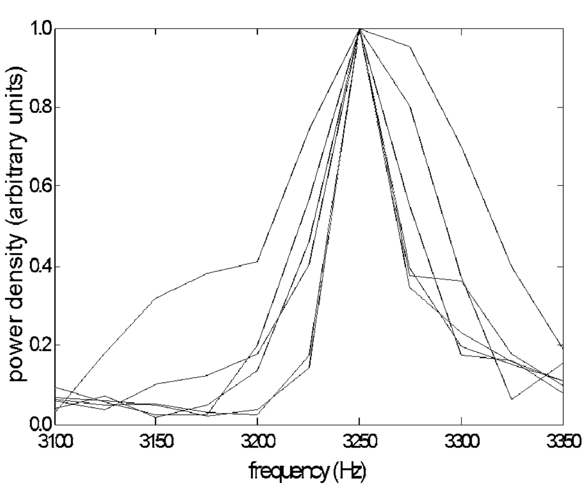

Fig. 7. Successive peaks of a 3250 Hz sinusoidal signal with different initial displacements of the loudspeaker. Peaks correspond to displacements of 5, 4, 3, 2, and 1 cm starting from the outermost peak to the innermost peak. Greater initial displacements result in larger spectral broadening.

can estimate the rod vibration frequency using the value of $B = 0.8306$ in Eq. (14). We substitute $c \approx 331$ m/s for the speed of sound and $f_0 = 3250$ Hz for the oscillation frequency and find that the estimated value of the rod vibration frequency is $\nu \approx 0.9$ Hz. This value is consistent with visual observation of the vibrating rod. However, no independent measurement of this frequency was done because of the need for instrumentation that is not available to us.

IV. EXPERIMENTAL DETAILS

A few experimental hints are worth mentioning here. Better results are obtained using a loudspeaker with a smaller diameter so that the point source becomes a better approximation. The experiment does not have to be done as we have described, where both data acquisition and analysis are carried out at the same time. Instead, the sound can be recorded on a hard disk using ordinary recording software and replayed at a later time for analysis. The analysis does not have to be done using Matlab and other data acquisition software can be used. Students are encouraged to write their own software using the language of their choice.

APPENDIX A: MATLAB M-FILE FOR AUDIO SPECTRAL ANALYSIS

The following software performs the following operations. (1) Acquire a preset number of data points from the computer sound card at a sampling rate of 8000 samples per second (statements 1–8). The number of data points is set by the samples per trigger statement. The time taken by the process is this number divided by the sampling rate. (2) Perform a fast Fourier transformation on the data and calculate the spectral power density over 512 channels (statements 12-13). (3) Assign frequencies to these channels. The overall frequency range selected is 0–12 kHz (statement 14). (4) Select the frequency range to be plotted (statements 15–17). The software is programmed to normalize all peak heights to a maximum of unity to make comparisons easier.

```
function f = spectrometer1
ai = analoginput('winsound')
addchannel(ai,1:2);
ai.samplerate = 8000;
ai.samplespertrigger = 8000;
ai.triggertype = 'immediat';
start(ai);
[d,t] = getdata(ai);
u1 = d(:,2);
u2 = d(:,1)
y1 = u1;
y = fft(u1,512);
p = y.*conj(y);
f = (0:50:25600)*256/512;
ff = f(10:256);
cc = (p(10:256))/max(p(10:256));
plot(ff,cc)
delete(ai)
clear ai
```

Data acquisition

Fast Fourier transformation

Plotting

APPENDIX B: MATLAB M-FILE FOR PERFORMING THE DOPPLER BROADENING EXPERIMENT

This software is a modification of the audio spectral analysis program in Appendix A. A loop is added so that five successive measurements are made and the time between successive runs is determined by the variable spt to determine the number of data acquired in each run. The third modification selects the frequency window to be plotted. Here we use a window in the neighborhood of the 3250 Hz peak.

```
function f = spectrometer2(spt)
hold
for i = 1:5
ai = analoginput('winsound')
addchannel(ai,1:2);
ai.samplerate = 8000;
ai.samplespertrigger = spt;
ai.triggertype = 'immediat';
start(ai);
[d,t] = getdata(ai);
u1 = d(:,2);
u2 = d(:,1)
y1 = u1;
y = fft(u1,512);
p = y.*conj(y);
f = (0:50:15000)*256/512;
ff = f(125:135);
cc = (p(125:135))/max(p(125:135));
plot(ff,cc)
delete(ai)
clear ai
end
```


[1] T. J. Benskya and S. E. Frey, “Computer sound card assisted measurements of the acoustic Doppler effect for accelerated and unaccelerated sound sources,” Am. J. Phys. **69**, 1231–1236 (2001).

[2] D. Halliday and R. Resnick, *Physics I & II* (Wiley, NY, 1966), p. 1008.

[3] R. E. Scott, *Linear Circuits* (Addison-Wesley, Reading, MA, 1960), p. 717.

[4] Grant R. Fowles, *Analytical Mechanics* (Holt, Rinehart and Winston, 1962), p. 50.