utilizing the inherent ability of simple chaotic oscillators to generate very wide bandwidth waveforms directly and efficiently. With chaos control, we can produce a vast array of waveforms that are ideal for spread-spectrum communications and for highly accurate ranging. We believe a system designed around a chaotic source can perform comparably to conventional systems while requiring far fewer components. The idea of exploiting the complexity of chaos in order to simplify hardware is the crux of chaos engineering. For these applications, the oscillators and controllers must operate at frequencies at and above several hundred megahertz. To control chaos at these speeds, we certainly cannot employ digital processors or complex analog circuitry to calculate the requisite control perturbations: the extremely short latency demanded of the feedback loop necessitates a simple controller. Besides such practical concerns, the true efficiency of chaos control is suspect if the controller is considerably more complex than the system under control. Hence, we believe the development of simple control techniques—such as limiter control and dynamic limiting—is essential for the development of practicable chaos engineering.

In summary, we have described a circuit for generating chaos along with an active limiter that can be used to stabilize periodic states using efficient chaos control. We have used this limiter to stabilize period-1 and period-2 orbits in the circuit, and have shown that the magnitudes of the control perturbations are minimal when controlling these natural states of the system. We then indicated how this simple control scheme can be extended to control higher order UPOs and arbitrary chaotic waveforms using a dynamic limiter. Due to its simple construction and operation, this system is well suited for laboratory demonstrations of chaos control that can provide students with exposure to the emerging field of chaos engineering.

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A computer-assisted experiment for the measurement of the temperature dependence of the speed of sound in air

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The measurement of the speed of sound in air is a classic experiment in introductory physics laboratory courses. Usually, the experiment is performed at room temperature. Here, we describe an experiment for measuring the speed of sound in air for temperatures between 15 and 65 °C. A typical lab session involving five temperatures can be performed in about 1 h. The collected data can also be used to obtain the ratio of specific heats for air, $\gamma \equiv c_P/c_V$.

The speed of sound, c_s, is related to the frequency ν and wavelength λ of a sound wave through the general relation $c_s = \nu\lambda$, where ν and λ are usually determined by considering the acoustic resonances of a cylindrical tube such as a resonance tube or Kundt's tube. In particular, for a tube of length L, closed at both ends, the resonance or standing wave condition requires $2L = n\lambda$, where n is an integer, $n = 1,2,3...$, and therefore the resonance frequencies ν_n are given by

$$\nu_n = \frac{c_s}{2L} n. \tag{1}$$

Thus, from the measurement of ν_n one can obtain the speed of sound c_s. In a hypothetical experiment, sound waves are produced by a small loudspeaker that is driven by a signal generator and placed at one end of the tube. A microphone located at the other end of the tube is connected to an oscilloscope or to an external loudspeaker. By using a fine tuning control, the frequency of the signal generator is increased slowly until a resonance condition is met. Under this condition, a relative maximum amplitude on the oscilloscope screen is observed or a relative maximum intensity of sound in the external loudspeaker is heard. Alternatively, one can fix the frequency of the sound wave and vary the length of the tube, by placing the microphone in a movable piston or by using two telescoping tubes, until a resonance condition is achieved.

For temperatures close to room temperature, the speed of sound in air is known to vary linearly with its Celsius temperature t according to the following relation:[1]

$$c_s = a + b\,t, \tag{2}$$

where $a = 331.4$ m/s is the speed of sound in air at 0 °C and $b = 0.61\ \mathrm{m\,s^{-1}\,°C^{-1}}$. Equation (2) can be obtained from

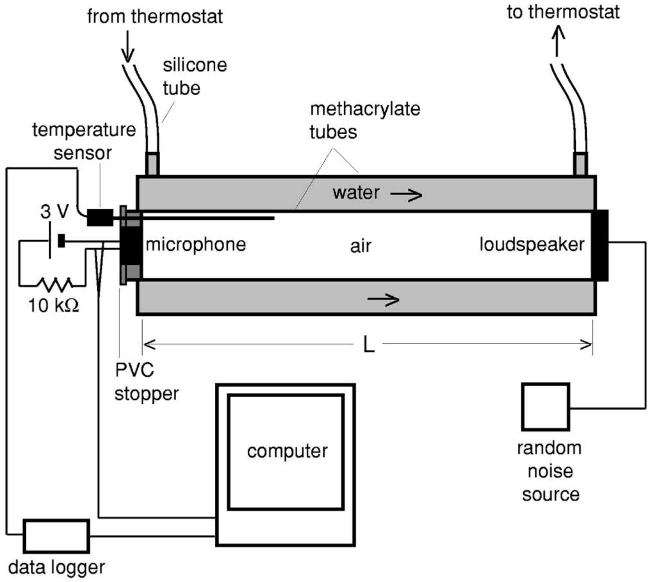

Fig. 1. The experimental setup includes the resonance tube, a thermostat, a data logger attached to a thermocouple, a personal computer, and an FM tuner.

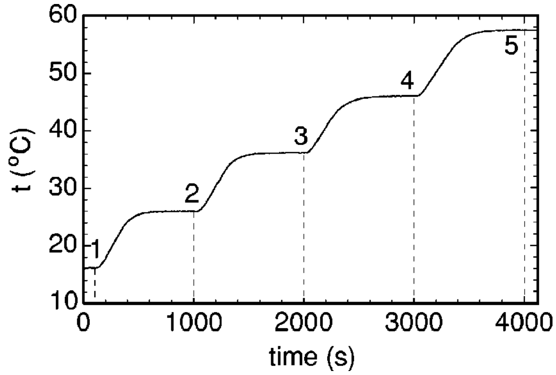

Fig. 2. Temperature of the air inside the resonance tube as time progressed during the experiment. Vertical dashed lines indicate the times when the resonance curves were captured.

simple thermodynamic considerations. The speed of a sound wave in a fluid is related to thermodynamic quantities by[2]

$$c_s = \sqrt{\frac{\gamma}{\rho \kappa_T}}, \tag{3}$$

where $\gamma \equiv c_P / c_V$ is the ratio of specific heats at constant pressure and constant volume, ρ is the mass density, and $\kappa_T \equiv -V^{-1}(\partial V/\partial P)_T$ is the isothermal compressibility of the fluid. For an ideal gas, Eq. (3) becomes

$$c_s = \sqrt{\frac{\gamma R T}{M}}, \tag{4}$$

where T is the absolute temperature, R is the universal gas constant, and M is the molar mass of the gas. If one takes $T = T_0 + t$, with $T_0 = 273.15$ K, and then expands Eq. (4) in a power series of t, truncating to first order, one obtains

$$c_s = \left(\frac{\gamma R T_0}{M}\right)^{1/2} + \frac{1}{2}\left(\frac{\gamma R}{M T_0}\right)^{1/2} t. \tag{5}$$

Taking $\gamma = 1.4$, $R = 8.314\ \mathrm{J\,mol^{-1}\,K^{-1}}$, and $M = 28.95 \times 10^{-3}$ kg/mol, Eq. (5) evaluates to become Eq. (2).

Equation (2) is usually employed to check the values of c_s obtained in laboratory experiments. Corrections due to humidity are usually neglected. Experiments specifically designed for measuring the temperature dependence of the speed of sound in air are not typical in physics labs. Ouseph and Link[3] have measured the variation of speed of sound in air with temperature, using Polaroid's ultrasonic ranging system. Bretz *et al.*[4] have proposed a spherical acoustic resonator that can be used to illustrate the temperature and pressure dependence of the speed of sound in gases. Here, we propose an apparatus based on Kundt's tube that allows one to perform the experiment in a very reasonable time.

In our experimental setup, two methacrylate tubes are used. One of them has a length of ~46 cm, an outside diameter of 4.9 cm, and a wall thickness of 0.4 cm. The other has a length of ~48 cm, an outside diameter of 2.5 cm, and a wall thickness of 0.2 cm. As shown in Fig. 1, the tubes are mounted coaxially by means of two plastic walls placed at the ends of the external tube. This tube has two holes that

Table I. The resonance frequencies obtained in our experiment for a tube of length $L = 45.0 \pm 0.1$ cm at five different temperatures. The frequencies are measured with an uncertainty of ± 3 Hz. For each temperature, the speed of sound (given in the last row) was obtained by equating $c_s/2L$ to the slope of the corresponding straight-line fit.

		t(°C)				
	n	16.2±0.1	26.4±0.1	36.6±0.2	47.4±0.2	57.7±0.3
	2	781	794	801	822	833
	3	1150	1171	1188	1205	1239
	4	1518	1548	1574	1610	1633
	5	1897	1926	1960	1999	2038
ν_n (Hz)	6	2277	2320	2352	2404	2444
	7	2656	2697	2750	2793	2843
	8	3036	3074	3136	3198	3255
	9	3404	3468	3528	3587	3654
	10	3783	3851	3909	3986	4065
	11	4169	4234	4313	4392	4471
	c_s (m/s)	341.2±1.1	346.8±1.1	352.9±1.1	359.5±1.1	366.2±1.1

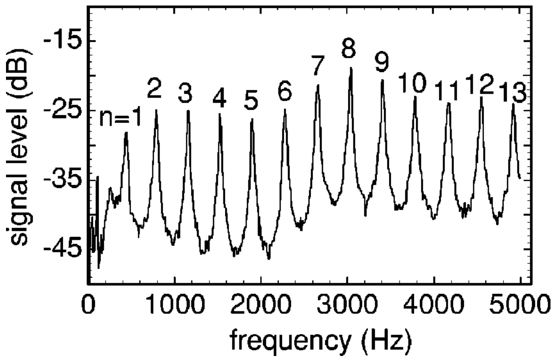

Fig. 3. Microphone spectrum obtained at $t_1 = 16.2\,°\mathrm{C}$ showing the various tube resonances. Each peak is labeled by its value of the integer n.

allow the entrance and the exit of water coming from a thermostat. A small loudspeaker with an impedance of 33 Ω and frequency response of 20–10 000 Hz is placed at one end of the internal tube (resonance tube). An electret condenser microphone is mounted inside a PVC or cork stopper and placed at the other end of the internal tube; the distance between the loudspeaker and the microphone is $L = 45.0 \pm 0.1$ cm. The microphone is powered by a 3 V battery. The loudspeaker is connected to a random noise generator. In our case, for simplicity, the noise source is an FM tuner that is set far away from any broadcast station. The microphone is connected to a computer, and its response is treated by means of standard real-time spectrum-analyzer software.[5] The temperature inside the internal tube is measured with a calibrated type K thermocouple. This thermocouple is also connected to the computer by using a data logger[6] so that the temperature is recorded throughout the experiment as shown in Fig. 2. The data logger is, of course, not essential and can be replaced by a standard temperature sensor.

The thermostat is started at room temperature, and the random noise source is then switched on. The noise can be assumed to have a continuous spectrum of frequencies and the tube acts like a filter, selecting its resonance frequencies. The signal received by the microphone is analyzed by the spectrum-analyzer software providing the *resonance curve* of the tube. This resonance curve is visible on the computer display screen and can be saved to disk in order to record the resonance frequencies. A typical result is shown in Fig. 3. After capturing the resonance curve, the noise source is switched off and the temperature of the thermostat is increased $\sim 10\,°\mathrm{C}$. We wait until the temperature of the air inside the resonance tube stabilizes at a new value. For our setup, about 15 min is required for stabilization. During this time the resonance frequencies of the preceding resonance curve are determined by simply positioning the mouse pointer over the peaks. In order to avoid the influence of a spurious peak that appears in the low frequency region, as can be seen in Fig. 3, the resonance frequencies are recorded from $n = 2$ to $n = 11$ only. When the system is ready for the new temperature, the noise source is switched on again and the above process is repeated. Figure 2 shows the time evolution of the air temperature inside the resonance tube, indicating the times when the five measurements were performed.

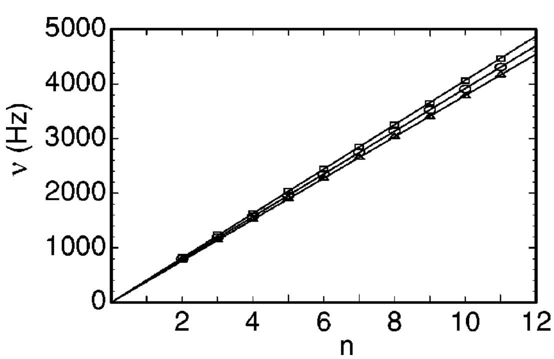

Fig. 4. Plot of resonance frequencies ν_n as a function of n for temperatures of 16.2 °C (open triangle), 36.6 °C (open ellipse), and 57.7 °C (open rectangle). Error bars are smaller than the symbol size. Solid lines give the corresponding best straight-line fit.

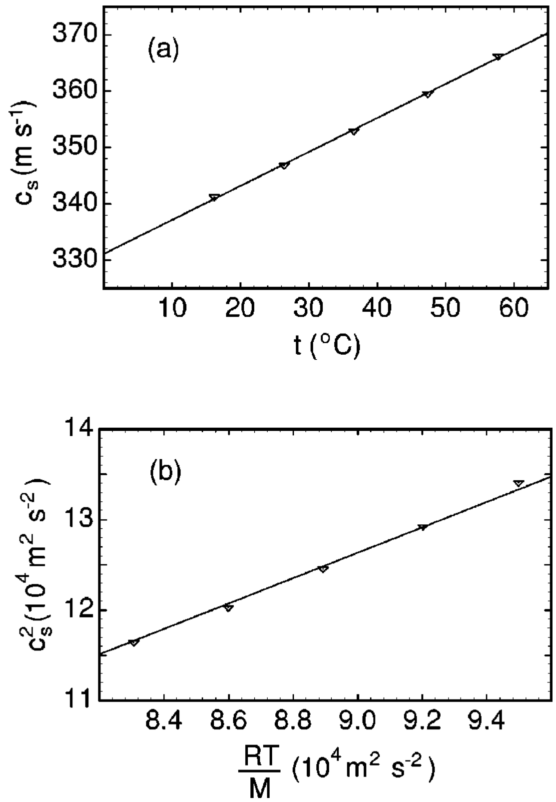

Fig. 5. (a) The speed of sound c_s given in the last row of Table I as a function of the Celsius temperature t. Solid line gives the best straight-line fit. (b) Plot of c_s^2 as a function of RT/M. Solid line gives the best straight-line fit. Error bars have a size comparable to that of the symbols.

A typical set of experimental results obtained by this procedure is shown in Table I. Frequencies are given with an uncertainty of ± 3 Hz. The resonance frequencies ν_n can be plotted versus n and fitted with a straight line whose slope yields $c_s/2L$. This is shown in Fig. 4 for three temperatures along with the corresponding best linear fit. The results obtained for c_s for all five temperatures are given in the last row of Table I. In Fig. 5(a), our measured speed of sound in air is plotted against Celsius temperature. The best linear fit is given by

$$c_s(\mathrm{m/s}) = (331 \pm 3\ \ \mathrm{m/s}) + \left(0.60 \pm 0.06 \frac{\mathrm{m/s}}{°\mathrm{C}}\right) t. \qquad (6)$$

This result is in excellent agreement with Eq. (2).

The value of γ may be determined by plotting c_s^2 vs RT/M, and considering a linear fit with y intercept equal to zero. This is shown in Fig. 5(b), where the fitted straight line has a slope of $\gamma = 1.40 \pm 0.01$.

In summary, we have designed a simple apparatus with which one may illustrate the temperature dependence of the speed of sound in air. Excellent results are obtained for both c_s and γ. The resonance tube that is used is inexpensive (including the loudspeaker and the microphone) and easy to construct. The experiment can be completed in about 1 h, a reasonable interval for a lab experiment or even for a classroom demonstration.

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[6]We used the TC-08 temperature logger produced by Pico Technology Ltd.

Visibility of thin-film interference fringes

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The visibility of fringes in a thin-film interference pattern depends on the reflectance of the film interfaces. The fringe visibility, a measure of the contrast between adjacent maxima and minima, is given by[1,2]

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}, \tag{1}$$

where $I_{\max}$ and $I_{\min}$ represent the intensity of the maxima and minima, respectively. For light incident along the normal, the visibility of the transmitted fringes, V_t, can be expressed as[3]

$$V_t = \frac{2R}{1 + R^2}, \tag{2}$$

where R is the reflectance of the film surfaces. Because the reflectance of a thin film surface must be less than one, this equation shows that increasing the reflectance of the interfaces will increase the visibility of the fringes in transmitted light.

This can be demonstrated with the following simple experiment. First we trap air between two plates of glass, and then between two one-way mirrors. In each case a spectrometer is used to record the transmitted light under white light illumination. A comparison is then made of the visibility of the resulting fringes for each experiment. It will be shown that using one-way mirrors dramatically increases the fringe visibility with respect to using glass plates.

Our apparatus is shown in Fig. 1. The light source S is a 60-W incandescent lamp with a clear glass tubular envelope and a linear filament.[4] Mirrors M_1 and M_2, which we purchase from a local supplier of plate glass and mirrors, are inexpensive and semi-transparent (partially silvered). Office supply binder clips are used to clamp them face-to-face. The mirrors are cleaned before placing their silvered surfaces together, and a similar arrangement is used for the pieces of plate glass. The spectra presented here are measured with an Ocean Optics USB2000 Fiber Optic Spectrometer,[5] which serves as the detector (D). The aperture (A) is 2 cm from the source; the mirror surfaces are 32 cm from the aperture.

The fact that higher reflectance leads to greater fringe visibility can be seen by comparing transmission spectra for glass plates and one-way mirrors. Spectra of light transmitted through thin air films are shown in Fig. 2. Spectrum A is from clear glass plates, whereas spectrum B is from one-way mirrors on tinted glass. The fringe visibility of the light transmitted through the thin film of air between glass plates is 0.10 while the visibility with the mirrors is 0.36. These

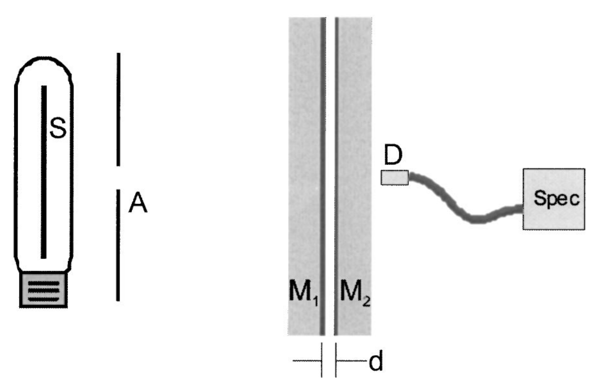

Fig. 1. Schematic diagram of apparatus: S—60-W incandescent tubular bulb, A—9-mm aperture in aluminum sheet metal, M_1 and M_2—glass plates/front surface half-silvered mirrors, d—air film thickness, D—fiber optic detector leading to CCD spectrometer (Spec).