

Fundamental Equations

Schrödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

Time-independent Schrödinger equation:

$$H\psi = E\psi, \quad \Psi = \psi e^{-iEt/\hbar}$$

Hamiltonian operator:

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V$$

Momentum operator:

$$\mathbf{p} = -i\hbar \nabla$$

Time dependence of an expectation value:

$$\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar} \langle [H, Q] \rangle + \left\langle \frac{\partial Q}{\partial t} \right\rangle$$

Generalized uncertainty principle:

$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [A, B] \rangle \right|$$

Heisenberg uncertainty principle:

$$\sigma_x \sigma_p \geq \hbar/2$$

Canonical commutator:

$$[x, p] = i\hbar$$

Angular momentum:

$$[L_x, L_y] = i\hbar L_z, \quad [L_y, L_z] = i\hbar L_x, \quad [L_z, L_x] = i\hbar L_y$$

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Fundamental Constants

Planck's constant:	$\hbar = 1.05457 \times 10^{-34} \text{ J s}$
Speed of light:	$c = 2.99792 \times 10^8 \text{ m/s}$
Mass of electron:	$m_e = 9.10938 \times 10^{-31} \text{ kg}$
Mass of proton:	$m_p = 1.67262 \times 10^{-27} \text{ kg}$
Charge of proton:	$e = 1.60218 \times 10^{-19} \text{ C}$
Charge of electron:	$-e = -1.60218 \times 10^{-19} \text{ C}$
Permittivity of space:	$\epsilon_0 = 8.85419 \times 10^{-12} \text{ C}^2/\text{J m}$
Boltzmann constant:	$k_B = 1.38065 \times 10^{-23} \text{ J/K}$

Hydrogen Atom

Fine structure constant:	$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = 1/137.036$
Bohr radius:	$a = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{\alpha m_e c} = 5.29177 \times 10^{-11} \text{ m}$
Bohr energies:	$E_n = -\frac{m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2 n^2} = \frac{E_1}{n^2} \quad (n = 1, 2, 3, \dots)$
Binding energy:	$-E_1 = \frac{\hbar^2}{2m_e a^2} = \frac{\alpha^2 m_e c^2}{2} = 13.6057 \text{ eV}$
Ground state:	$\psi_0 = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$
Rydberg formula:	$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$
Rydberg constant:	$R = -\frac{E_1}{2\pi\hbar c} = 1.09737 \times 10^7 / \text{m}$

Mathematical Formulas

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

Integrals:

$$\int x \sin(ax) dx = \frac{1}{a^2} \sin(ax) - \frac{x}{a} \cos(ax)$$

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax)$$

Exponential integrals:

$$\int_0^{\infty} x^n e^{-x/a} dx = n! a^{n+1}$$

Gaussian integrals:

$$\int_0^{\infty} x^{2n} e^{-x^2/a^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^{\infty} x^{2n+1} e^{-x^2/a^2} dx = \frac{n!}{2} a^{2n+2}$$

Integration by parts:

$$\int_a^b f \frac{dg}{dx} dx = - \int_a^b \frac{df}{dx} g dx + fg \Big|_a^b$$