

Mid-Term 2

Quantum Mechanics I
Physics 342

Date: April 4th, 2008

NAME:

There are three problems, each worth 10 points. Show as much work as you can for all problems. You have 50 minutes, and can use the front and back of a 3×5 (inch) card – the front and back covers of the book are provided separately.

Problem 2.1

(10 points, 3.3 per part)

- a. Find the Fourier transform of the function: $f(x) = e^{-|x|}$.
- b. What is the Hermitian conjugate of the operator $\hat{P} = i$ (think of \hat{P} as a function of x , and use the integral form of Hermiticity).

- c. The Legendre polynomials are orthonormal: $\int_{-1}^1 P_\ell(x) P_{\ell'}(x) dx = \frac{2}{2\ell+1} \delta_{\ell\ell'}$. Use this, and the first two polynomials $P_0(x) = 1$, $P_1(x) = x$ to construct $P_2(x)$ (hint: Start with the most general quadratic: $\bar{P}_2(x) = ax^2 + bx + c$ and apply the orthogonality conditions to fix (a, b, c)).

Problem 2.2

(10 points) Find the stationary states and allowed energies for the two-dimensional “infinite rectangular box” potential:

$$V(x, y) = \begin{cases} 0 & 0 < x < a \text{ and } 0 < y < b \\ \infty & \text{otherwise} \end{cases} \quad (1)$$

Problem 2.3

For the following initial wavefunction of Hydrogen¹:

$$\psi(r, \theta, \phi) = \frac{1}{8a\sqrt{\pi a^3}} r e^{-\frac{r}{2a}} e^{-i\phi} \sin \theta, \quad (2)$$

- a. Write the time-dependent solution $\Psi(\mathbf{r}, t)$.

¹Note that in terms of the Bohr radius a , the Hydrogen potential can be written as $V(r) = -\frac{\hbar^2}{m a r}$.

- b.** What values could you get from an energy measurement?